Advanced Placement Calculus AB

SD

.37

Year	5	4	3	2	1	T	Ave
03	15	12	4	1	1	33	4.18
02	15	5	2	2		24	4.38
01	13	8	5	1		27	4.22
00	8	4	6	3	1	22	3.68
99	8	3	2			13	4.46
98	8	4	3	1		16	4.19
97	10	5	1	4	1	21	3.90
96	9	3	6	1		19	4.05
95	6	4	5			15	4.07
94	9	8	1		1	19	4.26
93	7	5	7	2		21	3.81
92	4	7	3		2	16	3.69
91	3	7	11	4		25	3.36
90	8	3	1	1		13	4.38
89	5	5	7	1		18	3.78
88	6	5				11	4.54
87	4	6	3	1		14	3.21
86	12	7	2	3	1	25	4.00
85	7	3	9	5	1	25	3.40
84	6	6	5			17	4.06
83	5	9	6	4	1	25	3.38
82	5	6	1	1		13	4.15
81	7	5	4	2		18	3.94
80	3	7	3			13	4.0
Totals:	183	137	97	37	9	463	3.97
	39.5	29.6	20.9	8.0	1.9		

Do *not* settle for second best!

On problems 1, 2, 3 ONLY a graphing calculator is allowed.

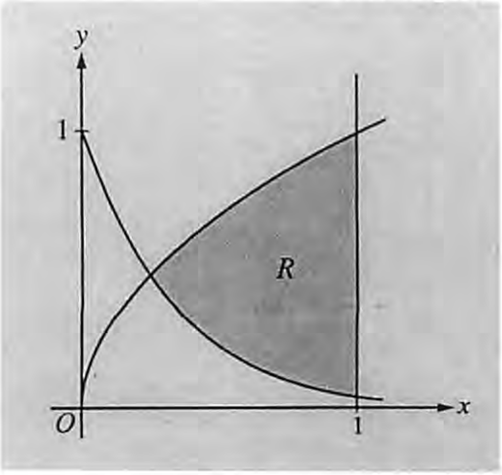
2003

1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure.

   a) Find the area of R.

   b) Find the volume of the solid generated when R is revolved about he horizontal line $y = 1$.

   c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.



2. A particle moves along the x-axis so that its velocity at time t is given by $v(t) = -(t+1)\sin\left(\dfrac{t^2}{2}\right)$.

   At time $t = 0$, the particle is at position $x = 1$.

   a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

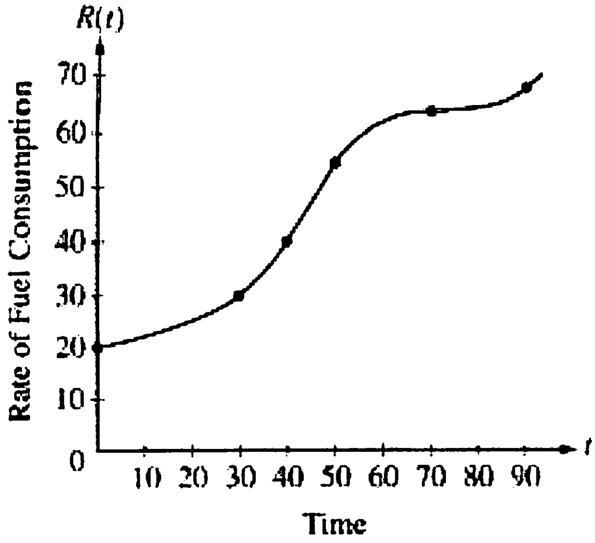
   b) Find all times t in the open interval $0 < t < 3$ when the particle changes directions. Justify your answer.

   c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

   d) During the time interval $0 \le t \le 3$. what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

On problems 1, 2, 3 ONLY a graphing calculator is allowed.

2003



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

3. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown.

   a) Use the data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

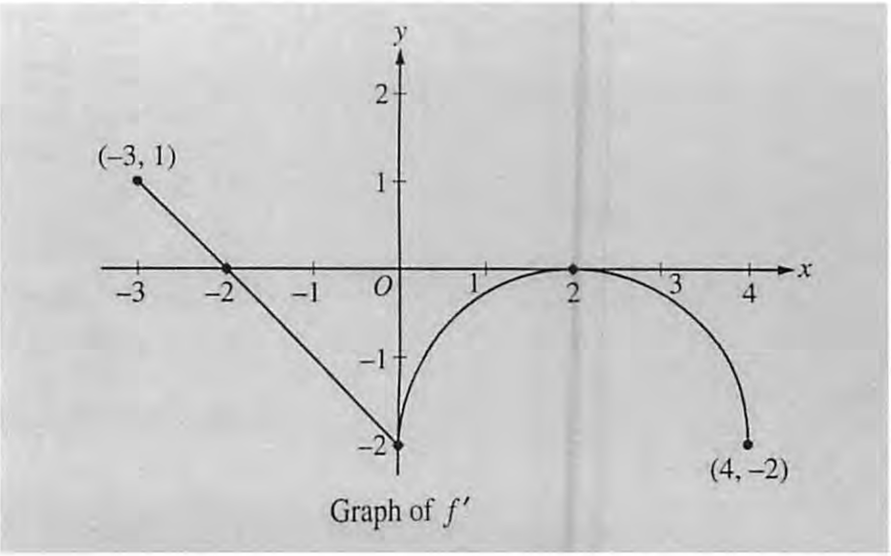
   b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

   c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

   d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

On problems 1, 2, 3 ONLY a graphing calculator is allowed.

2003



Graph of f'

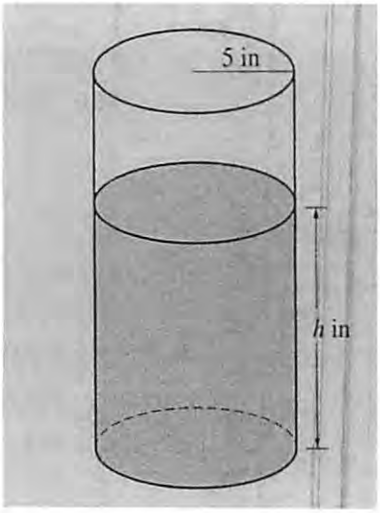
4. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as show above.

   a) On what intervals, if any, is f increasing? Justify your answer.

   b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

   c) Find an equation for the line tangent to the graph of f at the point (0 , 3).

   d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.



5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume of a cylinder with radius r and height h is $V = \pi r^2 h$.)

   a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$

   b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t.

   c) At what time t is the coffeepot empty?

On problems 1, 2, 3 ONLY a graphing calculator is allowed.

2003

6. Let f be the function defined by $f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5 \end{cases}$.

   a) Is f continuous at $x = 3$? Explain why or why not.

   b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

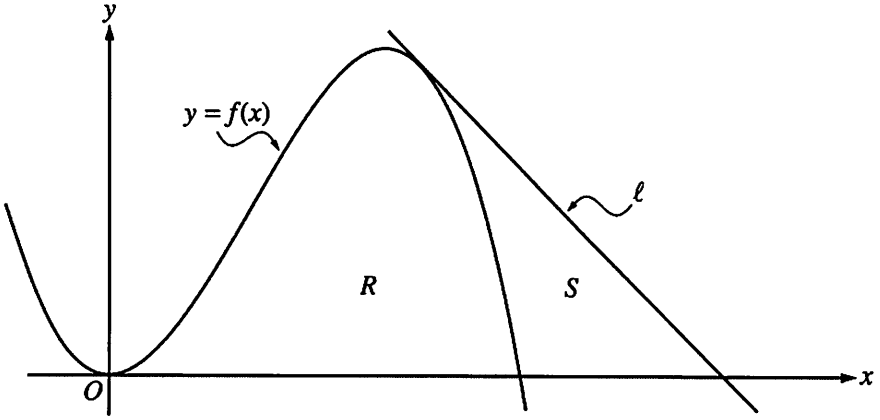
   c) Suppose the function g is defined by $g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5 \end{cases}$,

   where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m ?

CALCULUS AB
SECTION II, Part A

Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

   (a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when R is revolved about the x-axis.

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2. A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t + 1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

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Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

3. A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

   (a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.

   (b) Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

   (c) Using correct units, explain the meaning of $\pi\int_{125}^{275}\left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.

   (d) Explain why there must be at least one value x, for $0 < x < 360$, such that $B''(x) = 0$.

END OF PART A OF SECTION II

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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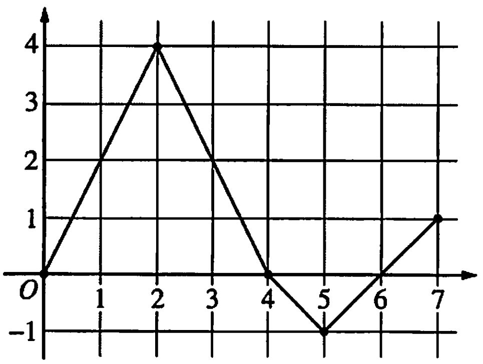
4. A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.
   (a) Find the acceleration of the particle at time $t = 3$.
   (b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.
   (c) Find all values of t at which the particle changes direction. Justify your answer.
   (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.



Graph of f

5. Let f be a function defined on the closed interval $[0, 7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.
   (a) Find $g(3)$, $g'(3)$, and $g''(3)$.
   (b) Find the average rate of change of g on the interval $0 \leq x \leq 3$.
   (c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b) ? Explain your reasoning.
   (d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

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6. Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25$.

   (a) Find $f''(3)$.

   (b) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.

END OF EXAMINATION

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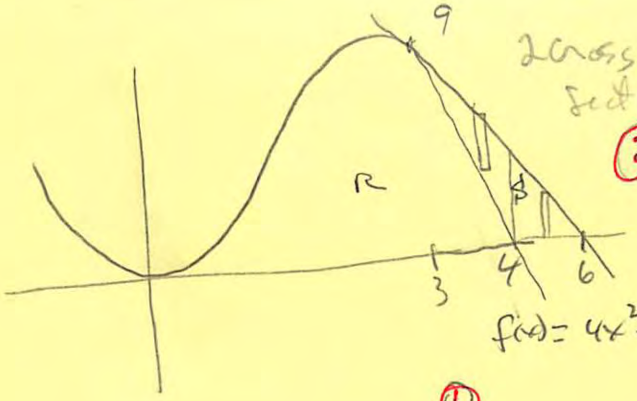


$f(x) = 4x^2 - x^3$

2 cross sections ②

$f(x) = x^2(4-x)$

$Y = -3x + 18$

2 integrals ①

③

①

A) $f'(x) = 8x - 3x^2 \overset{\text{set}}{=} -3$

$f(3) = 9$
$l(3) = 9$ Same pt.

$3x^2 - 8x - 3 = 0$
$(3x+1)(x-3) = 0$
$x \in \{-\frac{1}{3}, 3\}$.

② Since $x > 0$, $x = 3$
$3^2(4-3) = 9$
So $(3, 9)$ is the tangency coordinate.

⑦

B) $\int_3^4 -3x+18-(4x^2-x^3)\,dx +$

Proper integrals $\int_4^6 -3x+18\,dx$

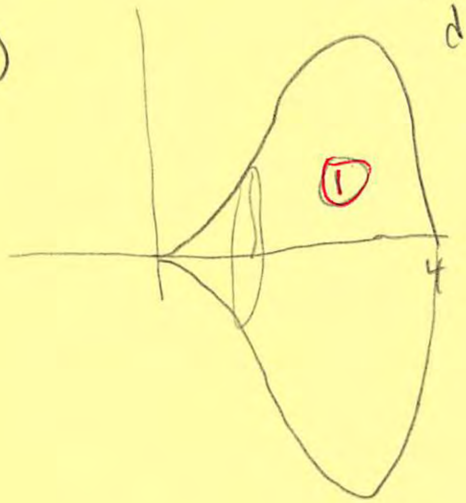
②

$1.9167 + 6 = \frac{95}{12}$
7.917

The area of Region S is 7.917

Answer/accurate
① ①

⑤ C)



"disks" ①

$\pi \int_0^4 (4x^2 - x^3)^2\,dx$ ②

①

$156.038\pi = 490.208$

$\frac{16384\pi}{105}$

②

$H(t) = 2 + \frac{10}{(1+\ln(t+1))}$ gal/hr. $T=0,\ V=125$

$R(t) = 12\sin\left(\frac{t^2}{47}\right)$ gal/hr.

3 a) $\int_0^{12} H(t)\,dt$ ① $= 70.571$ ① 70.571 Gal of oil ① are pumped into the tank during those 12 hrs.

3 b) If $H(t) - R(t) > 0$ ① The volume in the tank is rising else it is decreasing.

$H(6) = 5.395$ more oil is being consumed ① at $t=6$ So the level is decreasing.

① $R(6) = 8.319$

3 c) $125 + \int_0^{12} H(t)\,dt - \int_0^{12} R(t)\,dt = 122.026$ ① There are 122.026 Gal of oil in the tank at $t=12$

②

6 D) $v(t) = 125 + \int_0^{t} H(x)\,dx - \int_0^{t} R(x)\,dx$ ①

$v'(t) = \quad H(t) - R(t) \overset{\text{set}}{=} 0$ ①

$t = 11.318$ ①

$v''(11.318) = 5.221 > 0$ ∴ min at 11.318 f'' test.

①

②

③

x	0	60	120	180	240	300	360	
B(x)	24	30	28	30	26	24	26	← DiAmeter.

④ a) ① $\frac{1}{2}\cdot\frac{1}{360}\int_0^{360} B(x)\,dx$

↑ To create radius from DiAmeter

③ Proper average fctN,

③ B) ② $\left[120\left(30+30+24\right)\right]\frac{1}{720}$

$$\frac{\cancel{120}\cdot\overset{14}{\cancel{84}}}{\underset{\cancel{6}}{\cancel{720}}}$$

14 ①

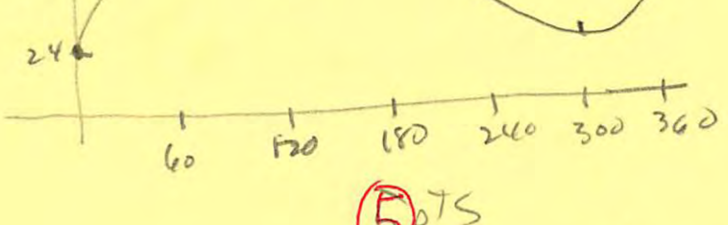
The average radius is 14 mm

③ c) $\pi\int_{125}^{275}\left(\frac{B(x)}{2}\right)^2 dx$

② {This is "disks" version which yields the volume of the Blood vessel from a point

① {125 mm from the start for a length of 150 mm.

⑤ D) B is given as twice differentiable so B is continuous and a smooth curve,



⑤ pts for argument.

B″(x) = 0 would occur if B had a change in concavity.

B appears to have 3 changes in concavity in the interval so there must be at least one point in 0 < x < 360 where B″(x) = 0

$v(t) = -1 + e^{1-t}$

③ A) $a(t) = v'(t)$ ①

$v'(3) = -e^{1-t}\Big|_{t=3}$

$= -e^{-2} = -\frac{1}{e^2}$ ②

④ B) $v(0) = -1 + e$

$v(1) = 0$

Speed $= |v(t)|$

$v(3) = -1 + e^{-2}$

$$v \quad \begin{array}{ccc} + & 0 & - \\ \hline \overset{\uparrow}{0} & 1 & \end{array}$$ ②

Speed $= v(t)$ if $0 \le t < 1$

Speed $= -v(t)$ if $t \ge 1$

$\frac{d(-v(t))}{dt} = +\frac{1}{e^t}$

Thus speed is increasing at $t = 3$. ②

③ C) The particle changes directions when $v(t)$ changes from + to − or vice versa → see v numberline

③ See v numberline

The only change is at $t = 1$

⑤ D) $\int_0^1 -1 + e^{1-t}\,dt + \int_1^3 1 - e^{1-t}\,dt$ ③

$-t - e^{1-t}\Big|_0^1 + t + e^{1-t}\Big|_1^3$

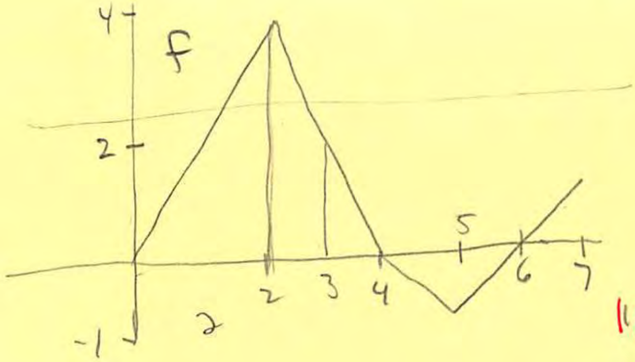
$-1 - 1 - (0 - e) + 3 + e^{-2} - (1 + 1)$

$-2 + e + 3 + e^{-2} - 2$

$-1 + e + \frac{1}{e^2}$ total distance

②

Speed increases when velocity AND acceleration are positive or when velocity AND acceleration are both negative.



⑥ a) $g(x) = \int_2^x f(t)\,dt$

1pt ea + 1pt for Reason ~~total~~ 6pts

$g(3) = \int_2^3 f(t)\,dt$

Trapazoid $\frac{1}{2} \cdot 1(4+2) = \underline{3}$

$\underline{g'(3) = f(3) = 2}$

$g''(3) = m$ of line at $x=3$

$(2,4) \quad (4,0)$

$\frac{0-4}{4-2} = \underline{-2 = g''(3)}$

③ b) average rate of change of g is ① average value of f

$\frac{1}{3}\int_0^3 f(t)\,dT =$

$\frac{1}{3}\left[g(3) - g(0)\right] =$

$\frac{1}{3}\left[3 - (-4)\right] \leftarrow$

$\frac{7}{3}$ ①

$g(0) = \int_2^0 f(t)\,dt$

$= -\int_0^2 f(t)\,dt$

$= -4$ ①

② c) Looking at the graph, ②

$g' = f = \frac{7}{3}$ for Two x values

between $x=0$ and $x=3$

④ D) g has an inflection point when $g' = f$ changes signs.

f' + u − u − u + u + ②

0 2 4 5 6 7

PI occur at $x = 2$ and $x = 5$

②

⑥

$f'(x) = \overset{u}{x}\,\overset{v}{\sqrt{f(x)}} \qquad f(3)=25$

⑧a) $f''(x) =$ ⑤ $\sqrt{f(x)} + \dfrac{x\,f'(x)}{2\sqrt{f(x)}}$ $\qquad f'(3) = 3\cdot 5$
$= 15$

$f''(3) = 5 + \dfrac{3\cdot \overset{3}{\cancel{15}}}{2\cdot\cancel{5}}$

$= 5 + \frac{9}{2}$ ③

$= \frac{19}{2}$.

⑦b) $\dfrac{dy}{dx} = x\sqrt{y}$

$\displaystyle\int \frac{dy}{\sqrt{y}} = \int x\,dx$ ②

$2\sqrt{y} = \frac{x^2}{2} + c$ ②

$2\sqrt{25} = \frac{9}{2} + c$

$10 - \frac{9}{2} = c$

$\frac{11}{2} = c$ ①

$2\sqrt{y} = \frac{x^2}{2} + \frac{11}{2}$

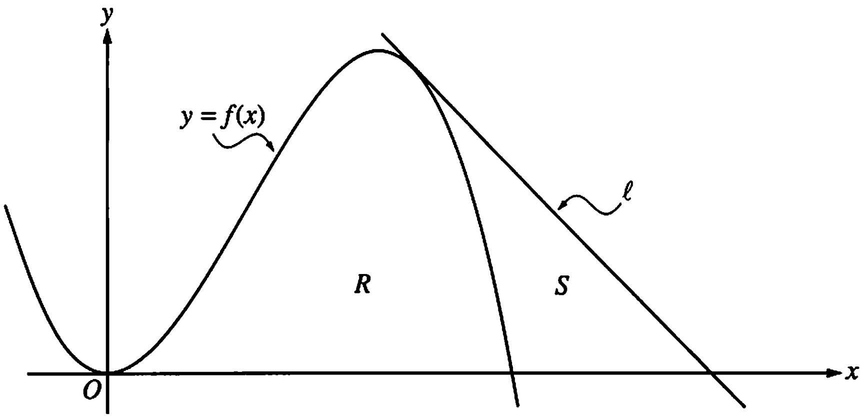
$= \frac{1}{2}(x^2+11)$

$\sqrt{y} = \frac{1}{4}(x^2+11)$

$y = \frac{1}{16}(x^2+11)^2$ ②

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

   (a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

   (b) Find the area of S.

   (c) Find the volume of the solid generated when R is revolved about the x-axis.

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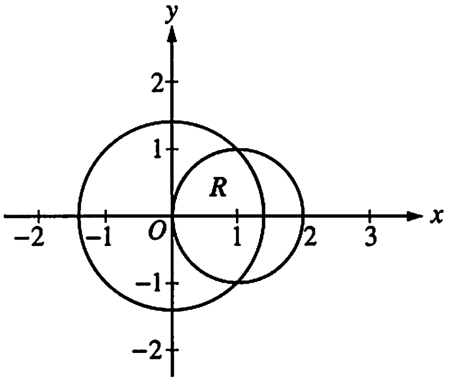

2. The figure above shows the graphs of the circles $x^2 + y^2 = 2$ and $(x - 1)^2 + y^2 = 1$. The graphs intersect at the points $(1, 1)$ and $(1, -1)$. Let R be the shaded region in the first quadrant bounded by the two circles and the x-axis.

   (a) Set up an expression involving one or more integrals with respect to x that represents the area of R.

   (b) Set up an expression involving one or more integrals with respect to y that represents the area of R.

   (c) The polar equations of the circles are $r = \sqrt{2}$ and $r = 2\cos\theta$, respectively. Set up an expression involving one or more integrals with respect to the polar angle θ R.

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

3. A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

   (a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.

   (b) Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

   (c) Using correct units, explain the meaning of $\pi\int_{125}^{275}\left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.

   (d) Explain why there must be at least one value x, for $0 < x < 360$, such that $B''(x) = 0$.

END OF PART A OF SECTION II

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CALCULUS BC
SECTION II, Part B

Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

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4. A particle moves in the xy-plane so that the position of the particle at any time t is given by

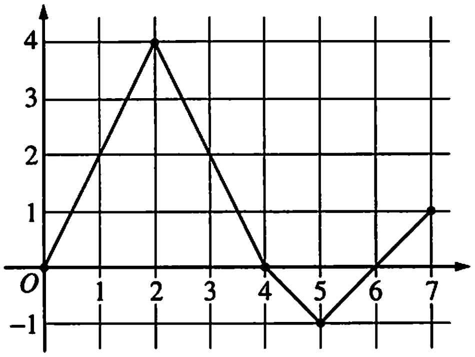
$$x(t) = 2e^{3t} + e^{-7t} \text{ and } y(t) = 3e^{3t} - e^{-2t}.$$

(a) Find the velocity vector for the particle in terms of t, and find the speed of the particle at time $t = 0$.

(b) Find $\frac{dy}{dx}$ in terms of t, and find $\lim\limits_{t \to \infty} \frac{dy}{dx}$.

(c) Find each value t at which the line tangent to the path of the particle is horizontal, or explain why none exists.

(d) Find each value t at which the line tangent to the path of the particle is vertical, or explain why none exists.



Graph of f

5. Let f be a function defined on the closed interval $[0, 7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\, dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b) ? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

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6. The function f has a Taylor series about $x = 2$ that converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x = 2$ is given by $f^{(n)}(2) = \frac{(n+1)!}{3^n}$ for $n \geq 1$, and $f(2) = 1$.

   (a) Write the first four terms and the general term of the Taylor series for f about $x = 2$.

   (b) Find the radius of convergence for the Taylor series for f about $x = 2$. Show the work that leads to your answer.

   (c) Let g be a function satisfying $g(2) = 3$ and $g'(x) = f(x)$ for all x. Write the first four terms and the general term of the Taylor series for g about $x = 2$.

   (d) Does the Taylor series for g as defined in part (c) converge at $x = -2$? Give a reason for your answer.

END OF EXAMINATION

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AB/BC 2

a) $P'(9) = -.646 \dashrightarrow P'(9) < 0$ So function is $\boxed{\text{not increasing}}$

b) $-\dfrac{d\,3e^{(-0.2\sqrt{x})}}{dx} = -3e^{(-0.2\sqrt{x})}\left(\dfrac{-0.2\sqrt{x}}{2\sqrt{x}}\right)$ which is positive for all $x > 0$

Thus $-3e^{(-0.2\sqrt{x})}$ is an increasing function for $x > 0$. Thus the absolute minimum value will happen at 0 or a critical point. $P'(0) < 0$, so the critical point where $p'(t) = 0$ is the absolute minimum value.

$p'(t) = 0$ for $t = \boxed{30.174}$

c) The integral of $P'(t)$ from 0 to t represents the amount of pollutant that has been added or subtracted from the lake over the period of time from 0 to t. You then add the pollutant that was present at t= 0 to get P(n).

$$50 + \int_0^{30.174} P'(t)\,dt = P(30.174) = 35.104 \dashrightarrow \boxed{\text{Yes}}$$

d) $P - 50 = -2(t - 0) \dashrightarrow P = -2t + 50$ For $P \le 40$ $\boxed{t \ge 5}$

AB/BC 4

a) $g(6) = 5 + \int_6^6 f(t)\,dt = 5 + 0 = \boxed{5}$ $g'(6) = f(6) = \boxed{3}$ $g''(6) = f'(6) = \boxed{0}$

b) f is the derivative of g, so where $f < 0$, g is decreasing.

$\boxed{[-3,0)(12,15]}$

c) $g'' < 0$ means g is concave down. $g'' = f'$ $f' < 0$ when f is decreasing

$\boxed{(6,15]}$

$$\frac{3}{2}[-1 + 2(0) + 2(1) + 2(3) + 2(1) + 2(0) + (-1)] = \boxed{12}$$

AB/BC 5

Note: $\int y\,dy = \int (3 - x)\,dx \dashrightarrow \dfrac{y^2}{2} = 3x - \dfrac{x^2}{2} + C$

a) If $y = -2$ is tangent to the graph, then the graph has 0 slope at this point.

$\dfrac{dy}{dx} = 0 = \dfrac{3 - x}{y}$ Thus $0 = \dfrac{3 - x}{-2} \dashrightarrow \boxed{x = 3}$

In a neighborhood of $y = -2$, $y \le 0$. For $x < 3$, $(3 - x) > 0$, so $\dfrac{3 - x}{y} < 0$

For $x > 3$, $(3 - x) < 0$, so $\dfrac{3 - x}{y} > 0$. This makes the point, $(3,-2)$ a

$\boxed{\text{relative minimum}}$

b) $\int y\,dy = \int (3 - x)\,dx \dashrightarrow \dfrac{y^2}{2} = 3x - \dfrac{x^2}{2} + C$ Using $g(6) = -4$

$\dfrac{16}{2} = 3(6) - \dfrac{36}{2} + C \dashrightarrow C = 8 \dashrightarrow \dfrac{y^2}{2} = 3x - \dfrac{x^2}{2} + 8 \dashrightarrow$

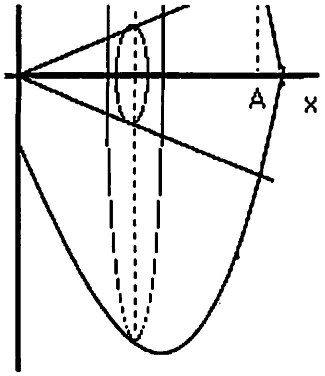
$y^2 = 6x - x^2 + 16 \dashrightarrow$ $\boxed{y = -\sqrt{6x - x^2 + 16}}$ (Note: Choose negative square root so that $g(6) = -4$.)

c) $\sum_{n=1} \frac{1}{n^{2p}}$ diverges if $2p \le 1 \dashrightarrow p \le \frac{1}{2}$ $\sum_{n=1} \frac{(-1)^n}{n^p}$ converges if $p > 0 \dashrightarrow$

Combining the conditions, $\boxed{0 < p \le \frac{1}{2}}$

d) $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ converges if $2p > 1 \dashrightarrow p > \frac{1}{2}$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ diverges if $p \le 1$ $\dashrightarrow$

Combining the conditions, $\boxed{\frac{1}{2} < p \le 1}$



b) $$\pi\int_0^A \left(\left(4x-x^3+1\right)^2-\left(\frac{3}{4}x\right)^2\right)dx=\boxed{57.463}$$

c) $$\boxed{1+\int_0^A\sqrt{1+\left(4-3x^2\right)^2}\,dx+\int_0^A\sqrt{1+\left(\frac{3}{4}\right)^2}\,dx}$$ Note: 1 is the y-axis part.

BC6

Note: $\sum_{n=1}^{\infty}\frac{x^n}{n}=x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\ldots$

a) $\ln\left(\frac{1}{1+3x}\right)=\ln\left(\frac{1}{1-(-3x)}\right)=$

$=-3x+\frac{9x^2}{2}-\frac{27x^3}{3}+\frac{81x^2}{4}-\frac{243x^2}{5}+\ldots\quad\sum_{n=1}^{\infty}\frac{(-3x)^n}{n}$

$\lim_{n\to\infty}\left|\frac{(-3x)^{n+1}}{n+1}\bullet\frac{n}{(-3x)^n}\right|=\lim_{n\to\infty}\left(\frac{n}{n+1}\right)|-3x|=|3x|$

For $|3x|<1\rightarrow -1<3x<1\rightarrow -\frac{1}{3}<x<\frac{1}{3}$

$x=-\frac{1}{3}\rightarrow\sum_{n=1}^{\infty}\frac{1}{n}$ diverges [p-series p=1] and $x=\frac{1}{3}\rightarrow\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ [alternating series with decreasing terms] converges, so interval of convergence is $\boxed{-\frac{1}{3}<x\le\frac{1}{3}}$

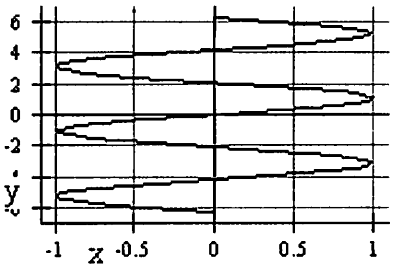
b) Using $\ln\left(\frac{1}{1-x}\right)=\sum_{n=1}^{\infty}\frac{x^n}{n}=x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\ldots$ for $-1\le x<1$,

use $x=-1$ So $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}=\ln\left(\frac{1}{1-(-1)}\right)=\boxed{\ln\left(\frac{1}{2}\right)}$

∞ ∞

Back to the index

BC1



a) ↑ is direction of motion along path.

List of values where $x = 0$ from bottom to top: $-2\pi, -\frac{4\pi}{3}, -\frac{2\pi}{3}, 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$

b) $\boxed{\text{Range of } x(t) = -1 \le x \le 1 \quad \text{Range of } y(t) = -2\pi \le y \le 2\pi}$

c) $x'(t) = 3\cos(3t)$ Set = 0 --> $3\cos(3t) = 0$ --> $t = -\frac{5\pi}{6}, -\frac{\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

t	+	0	−
x'(t)		$\frac{\pi}{6}$	

$\boxed{t = \frac{\pi}{6} \text{ smallest positive t that is a local maximum}}$

The speed of the particle is $\left|\vec{v}\right| = \sqrt{\left(x'\left(\frac{\pi}{6}\right)^2\right) + \left(y'\left(\frac{\pi}{6}\right)^2\right)} = \boxed{2}$

d) $5\pi \sim 15.708$ Use arc length $\int_{-\pi}^{\pi} \sqrt{x'(t)^2 + y'(t)^2}\, dt = 17.973$

$\boxed{\text{The distance traveled is greater than } 5\pi}$

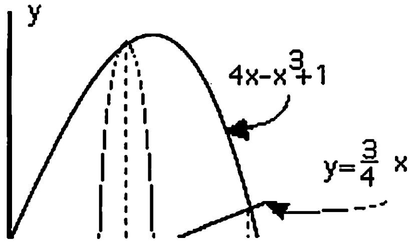
BC3

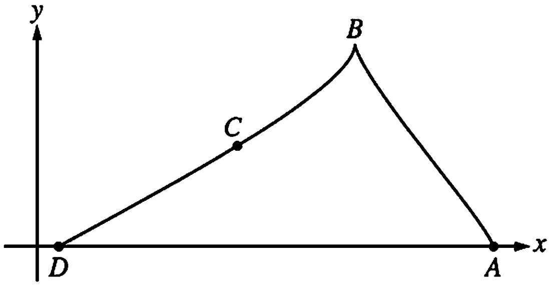
Note: Point of intersection of two curves is (1.940,1.455) Let A = 1.940

$y_1 = 4x - x^3 + 1 \qquad y_2 = \frac{3}{4}x$

$y_1 = 4x - x^3 + 1 \qquad y_2 = \frac{3}{4}x$

a) $y = 4x - x^3 + 1$ is the upper curve, so $\int_0^A \left(4x - x^3 + 1 - \frac{3}{4}x\right) dx = \boxed{4.515}$





2. A particle starts at point A on the positive x-axis at time $t = 0$ and travels along the curve from A to B to C to D, as shown above. The coordinates of the particle's position $(x(t), y(t))$ are differentiable functions of t, where $x'(t) = \frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\frac{\pi\sqrt{t+1}}{2}\right)$ and $y'(t) = \frac{dy}{dt}$ is not explicitly given. At time $t = 9$, the particle reaches its final position at point D on the positive x-axis.

   (a) At point C, is $\frac{dy}{dt}$ positive? At point C, is $\frac{dx}{dt}$ positive? Give a reason for each answer.

   (b) The slope of the curve is undefined at point B. At what time t is the particle at point B?

   (c) The line tangent to the curve at the point $(x(8), y(8))$ has equation $y = \frac{5}{9}x - 2$. Find the velocity vector and the speed of the particle at this point.

   (d) How far apart are points A and D, the initial and final positions, respectively, of the particle?

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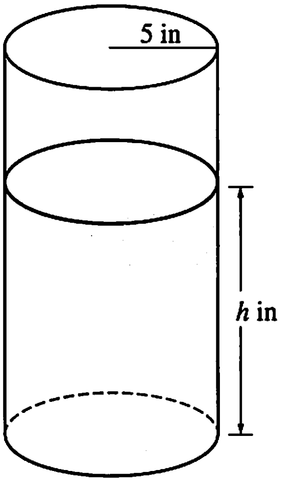

5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

   (a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

   (b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t.

   (c) At what time t is the coffeepot empty?

---

6. The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \cdots$$

   for all real numbers x.

   (a) Find $f'(0)$ and $f''(0)$. Determine whether f has a local maximum, a local minimum, or neither at $x = 0$. Give a reason for your answer.

   (b) Show that $1 - \frac{1}{3!}$ approximates $f(1)$ with error less than $\frac{1}{100}$.

   (c) Show that $y = f(x)$ is a solution to the differential equation $xy' + y = \cos x$.

END OF EXAMINATION

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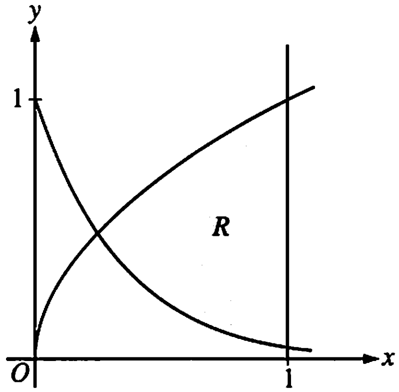
CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.
   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

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GO ON TO THE NEXT PAGE.

Jim Mumaugh - Vaca High
From: Jim [teeitup98@sbcglobal.net]
Sent: Tuesday, May 13, 2003 5:22 PM
To: Jim Mumaugh - Vaca High
Subject: FW: [ap-calculus] RE: Consensus Answers

------ Forwarded Message

From: **"Jennifer Nicholls" <J.Nicholls@email.bonita.k12.ca.us>**
Reply-To: "AP-Calculus" <ap-calculus@lyris.collegeboard.com>
Date: Tue, 13 May 2003 10:22:26 -0700
To: "AP-Calculus" <ap-calculus@lyris.collegeboard.com>
Subject: [ap-calculus] RE: Consensus Answers

Is #3b 0 or 1/20? I was thinking about approximating it by using the table, but it doesn't say approximate, so I was concerned about using a difference quotient twice. So instead I thought of it as the maximum of R'(t) at tE. Since R(t) is twice differentiable, then R'(t) is once differentiable. So the maximum must occur where the derivative of R'(t) = 0. That means that at R"(45)=0.

Is there an error in my thinking - is the consensus 1/20 and not 0?

Jennifer Nicholls
Bonita High School

-----Original Message-----
From: **Bob Enenstein [mailto:doctore98@yahoo.com]**
Sent: Monday, May 12, 2003 10:06 PM
To: AP-Calculus
Subject: [ap-calculus] Consensus Answers

After looking at other posting and doing the problems myself, here is my stab at a consensus set of answers ...

1. AB
a) .443 sq units
b) 1.424 cu units
c) 1.554 cu units

2. AB
a) a(2) = 1.588; decreasing
b) t = 2.507
c) 4.334
d) 2.265

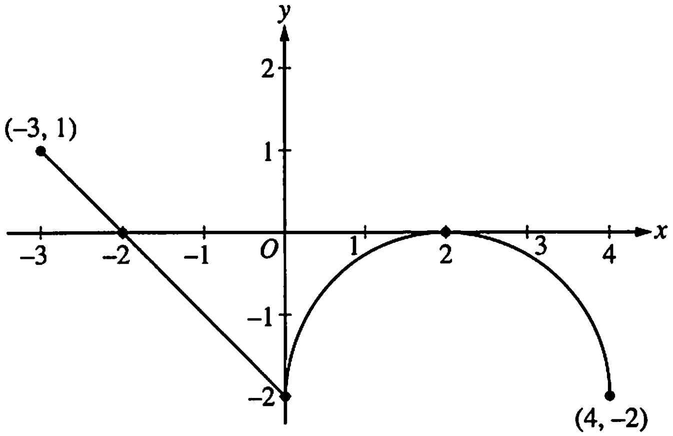
3. AB
a) 1.5 gal/min/min
b) R" = 1/20 gal/min/min/min
c) 3700 gallons less
d) gal of fuel consumed from t=0 min to t=b min;
average rate of fuel consumption in gal/min from t=0 to t=b min

4. AB

CALCULUS BC
SECTION II, Part B

Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f'

4. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

   (a) On what intervals, if any, is f increasing? Justify your answer.

   (b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

   (c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

   (d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

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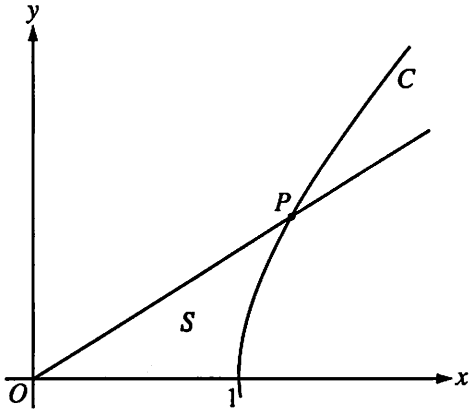

3. The figure above shows the graphs of the line $x = \frac{5}{3}y$ and the curve C given by $x = \sqrt{1 + y^2}$. Let S be the shaded region bounded by the two graphs and the x-axis. The line and the curve intersect at point P.

   (a) Find the coordinates of point P and the value of $\frac{dx}{dy}$ for curve C at point P.

   (b) Set up and evaluate an integral expression with respect to y that gives the area of S.

   (c) Curve C is a part of the curve $x^2 - y^2 = 1$. Show that $x^2 - y^2 = 1$ can be written as the polar equation $r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}$.

   (d) Use the polar equation given in part (c) to set up an integral expression with respect to the polar angle θ
   S.

END OF PART A OF SECTION II

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Subject: [ap-calculus] 2003 answers - first approximation
Date: Saturday, May 10, 2003 8:09 AM
From: Howard Jackson <hjackson@ahs.albany.k12.ny.us>
Reply-To: "AP-Calculus" <ap-calculus@lyris.collegeboard.com>
To: AP-Calculus <ap-calculus@lyris.collegeboard.com>

All mistakes are typo's ;-)

1. Both
a) .443
b)1.424
c)1.554

2 AB
a) a(2) = 1.588
decreasing
b) t = 2.507
c) 2.265

2 BC
a) No and No
b) t=3
c) v(8)=(-9/2,-5/2) |v(8)| = sqrt(106)/2
d) 49.907

3 AB
a) 1.5 gal/min^2
b) 0
c) 3700 less
d) gal of fuel consumed from t=0 to t=b
average rate of fuel consumption in gal/min from t=0 to t=b

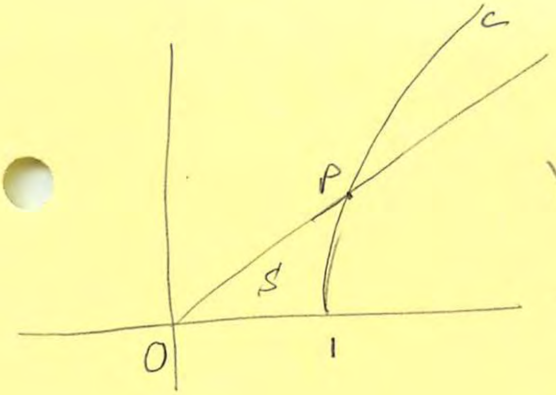
3 BC
a) (5/4,3/4) x'=3/5
b) ln(2)/2
c) proof
d) .5 Integral from 0 to inverseTan(3/5) of given r^2 dtheta

4. Both
a) [-3,2] Let the battle over endpoints begin!
b) 0 and 2
c) y = -2x + 3
d) f(-3)=9/2 f(4)=2pi-5

5 Both
a) proof
b) h=(sqrt(17)-t/10)^2
c) 10sqrt(17)

6 AB
a) yes
b) 4/3
c) m=2/5 k=8/5

6 BC
a) f'(0)=0 f"(0)=-1/3 max by 2nd deriv test
b) |error|<1/5!
c)proof

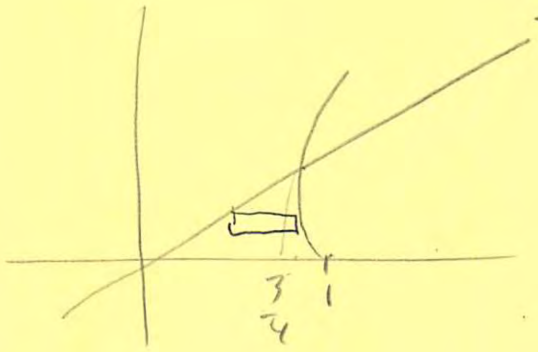


$y = \frac{3}{5}x$ $\quad x = \sqrt{1+y^2}$

$x = \frac{5}{3}y \quad \frac{3}{4}$ $\quad \frac{dx}{dy} = \frac{\cancel{2}y}{\cancel{2}\sqrt{1+y^2}}$

$\frac{dx}{dy} = \frac{5}{3}$

$\frac{dx}{dy} = \frac{y}{\sqrt{1+y^2}} = \cancel{.480}$



$\cancel{y = \frac{3}{5}\cdot\frac{3}{4} = \frac{9}{20}}$

$\frac{\frac{9}{20}}{\sqrt{\ }} = .6 = \frac{3}{5}$

$.75, .4$

$y = \frac{3}{4} \therefore x = \frac{5}{4}$

$$\int_0^{\frac{3}{4}} \sqrt{1+y^2} - \frac{5}{3}y \, dy = \cancel{.647}\ .347$$

c) $x^2 - y^2 = 1$

$r^2\cos^2\theta - r^2\sin^2\theta = 1$

$r^2(\qquad) =$

$r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}$

D) $\int \frac{r^2}{2} d\theta = \int_0^{\sin^{-1}\left(\frac{3}{5}\right)} \frac{1}{\cos^2\theta - \sin^2\theta} d\theta$

a) -3 < x < -2 (I'm from the increasing in a symmetric neighborhood requirement clan)
b) x = 0 and x=2 is when f'(x) changes sign
c) y = -2x + 3
d) f(-3)=4.5; f(4)=2 pi-5

5. AB
a) Set V = 25 pi * h and differentiate both sides w.r.to t
b) h=(sqrt(17)-t/10)^2
c) t□sqrt(17)

6. AB
a) Yes f(3) = 2, lim of f as x approaches 3 = 2 from left and right
b) average value = 4/3
c) m=2/5; k=8/5

1. BC
a) .443 sq units
b) 1.424 cu units
c) 1.554 cu units

2. BC
a) Negative since moving down and Negative since moving left
b) t=3
c) v(8)=<-9/2,-5/2>; |v(8)| = sqrt(106)/2 or 5.148
d) 39.255

3. BC
a) (5/4,3/4); x'=3/5
b).347
c) proof
d) .5 Integral from 0 to arctan(3/5) of 1/(cos^2(theta) - sin^2(theta)) d theta

4. BC
a) -3 < x < -2 (I'm from the increasing in a symmetric neighborhood requirement clan)
b) x = 0 and x=2 is when f'(x) changes sign
c) y = -2x + 3
d) f(-3)=4.5; f(4)=2 pi-5

5. BC
a) Set V = 25 pi * h and differentiate both sides w.r.to t
b) h=(sqrt(17)-t/10)^2
c) t□sqrt(17)

6 BC
a) f'(0)=0; f"(0)=-1/3; local max by 2nd deriv test
b) Decreasing alternating series so |error|<1/5!
c) proof

Jim Mumaugh - Vaca High
From: Jim [teeitup98@sbcglobal.net]
Sent: Tuesday, May 13, 2003 5:21 PM
To: Jim Mumaugh - Vaca High
Subject: FW: [ap-calculus] Consensus Answers

------ Forwarded Message

From: **Bob Enenstein <doctore98@yahoo.com>**
Reply-To: "AP-Calculus" <ap-calculus@lyris.collegeboard.com>
Date: Mon, 12 May 2003 22:06:23 -0700 (PDT)
To: "AP-Calculus" <ap-calculus@lyris.collegeboard.com>
Subject: [ap-calculus] Consensus Answers

After looking at other posting and doing the problems myself, here is my stab at a consensus set of answers ...

1. AB
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b) R" = 1/20 gal/min/min/min
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average rate of fuel consumption in gal/min from t=0 to t=b min

4. AB
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b) h=(sqrt(17)-t/10)^2
c) t□sqrt(17)

6. AB
a) Yes f(3) = 2, lim of f as x approaches 3 = 2 from left and right
b) average value = 4/3
c) m=2/5; k=8/5

1. BC
a) .443 sq units

b) 1.424 cu units
c) 1.554 cu units

2. BC
a) Negative since moving down and Negative since moving left
b) t=3
c) v(8)=<-9/2,-5/2>; |v(8)| = sqrt(106)/2 or 5.148
d) 39.255

3. BC
a) (5/4,3/4); x'=3/5
b).347
c) proof
d) .5 Integral from 0 to arctan(3/5) of 1/(cos^2(theta) - sin^2(theta)) d theta

4. BC
a) -3 < x < -2 (I'm from the increasing in a symmetric neighborhood requirement clan)
b) x = 0 and x=2 is when f"(x) changes sign
c) y = -2x + 3
d) f(-3)=4.5; f(4)=2 pi-5

5. BC
a) Set V = 25 pi * h and differentiate both sides w.r.to t
b) h=(sqrt(17)-t/10)^2
c) t□sqrt(17)

6 BC
a) f'(0)=0; f"(0)=-1/3; local max by 2nd deriv test
b) Decreasing alternating series so |error|<1/5!
c) proof

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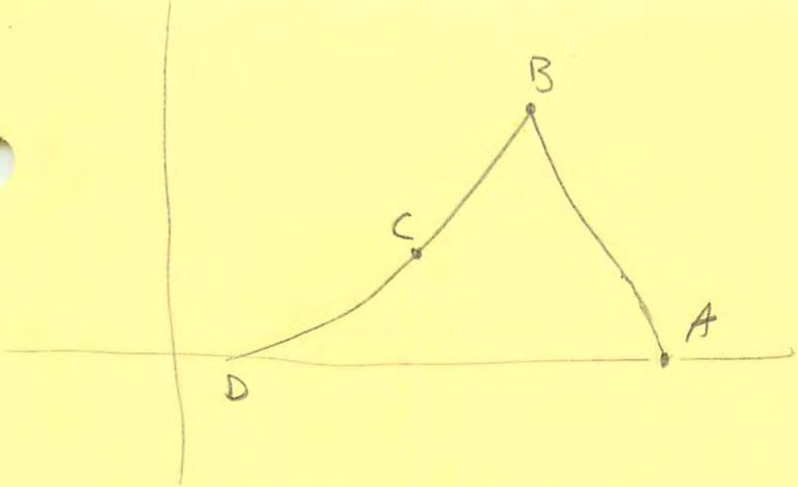

$(x(t), y(t))$

$x'(t) = \frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\frac{\pi\sqrt{t+1}}{2}$

$y'(t) = \frac{dy}{dt}$

a) at point C $\frac{dy}{dt}$ is negative because the y-value of the particle is decreasing.

at point C $\frac{dx}{dt}$ is decreasing because the particle is moving left.

b) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ is undefined when $\frac{dx}{dt} = 0$

$\frac{dy}{dt} = 0$ when $\cos\left(\frac{\pi t}{6}\right) = 0$ $t \in \{3, 9,$

$\frac{\pi}{6}t = \frac{\pi}{2}, \frac{3\pi}{2}$

$t = 3; 9$, etc.

$t = 3$

or $\frac{\cancel{\pi}\sqrt{t+1}}{2} = k\cancel{\pi}$

$\sqrt{t+1} = 2k$

$t+1 = 4k^2$

$t = 4k^2 - 1$

$k = 0$, $t = -1$

$k = \pm 1$ $t = 3$

$k = \pm 2$ $t = 15$,

c) $y = \frac{5}{9}x - 2$ $(x(8), y(8))$

College Board AP ADVANCED PLACEMENT PROGRAM®

CALCULUS AB
VACAVILLE HIGH SCHOOL

The following charts indicate the number of AP students from your school/district in each fifth of the total student group for each category described.

2002

AP GRADES	GRADE DISTRIBUTIONS GLOBAL	GRADE DISTRIBUTIONS GROUP
5	18.0%	62.5%
4	22.9%	20.8%
3	26.3%	8.3%
2	17.1%	8.3%
1	15.6%	0.0%
TOTAL STUDENTS	153966	24

MULTIPLE-CHOICE

	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN
DIFFERENTIAL CALCULUS (29 Q.)	1	2	1	3	17	29	11.1	18.7
INTEGRAL CALCULUS (16 Q.)		2	2	8	12	16	7.2	10.5
PART A- NO CALCULATOR (28 Q.)		3	1	1	19	28	11.0	19.6
PART B- WITH GRAPHING CALCULATOR (17 Q.)	1	1	6	10	6	17	7.3	9.7
WEIGHTED M-C SCORE	1	1	2	3	17	54	21.9	35.1
PERCENT IN EACH FIFTH	4%	4%	8%	13%	71%			

FREE-RESPONSE

	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN
AREA AND VOLUME-AB & BC	1	3	4	5	11	9	4.1	5.9
MODELING RATES-AB & BC	1	4	3	4	12	9	3.1	5.0
DISTANCE/VELOCITY/ACCELERATION	2	3	6	3	10	9	3.1	4.1
FTC WITH GRAPHICAL ANALYSIS-AB & BC	1	3	3	6	11	9	3.5	5.6
RELATED RATES	1	1	2	5	15	9	2.3	5.4
ANALYSIS OF FUNCTION DEFINED NUMERICALLY	2	2	3	7	10	9	2.3	3.8
WEIGHTED F-R SCORE	1	2	3	4	14	54	18.5	29.9
PERCENT IN EACH FIFTH	4%	8%	13%	17%	58%			

AI053595
VACAVILLE HIGH SCHOOL

100 MONTE VISTA AVENUE
VACAVILLE CA 95688

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Advanced Placement Calculus AB

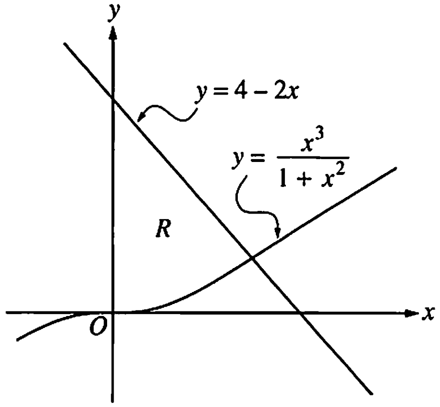
Year	5	4	3	2	1	T	Ave
02	15	5	2	2		24	4.38
01	13	8	5	1		27	4.22
00	8	4	6	3	1	22	3.68
99	8	3	2			13	4.46
98	8	4	3	1		16	4.19
97	10	5	1	4	1	21	3.90
96	9	3	6	1		19	4.05
95	6	4	5			15	4.07
94	9	8	1		1	19	4.26
93	7	5	7	2		21	3.81
92	4	7	3		2	16	3.69
91	3	7	11	4		25	3.36
90	8	3	1	1		13	4.38
89	5	5	7	1		18	3.78
88	6	5				11	4.54
87	4	6	3	1		14	3.21
86	12	7	2	3	1	25	4.00
85	7	3	9	5	1	25	3.40
84	6	6	5			17	4.06
83	5	9	6	4	1	25	3.38
82	5	6	1	1		13	4.15
81	7	5	4	2		18	3.94
80	3	7	3			13	4.0
Totals:	168	125	93	36	8	430	3.95

Do *not* settle for second best!

NAME

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the y-axis and the graphs of $y = \frac{x^3}{1 + x^2}$ and $y = 4 - 2x$, as shown in the figure above.
   (a) Find the area of R.
   (b) Find the volume of the solid generated when R is revolved about the x-axis.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

---

2. The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.
   (a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?
   (b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.
   (c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.
   (d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

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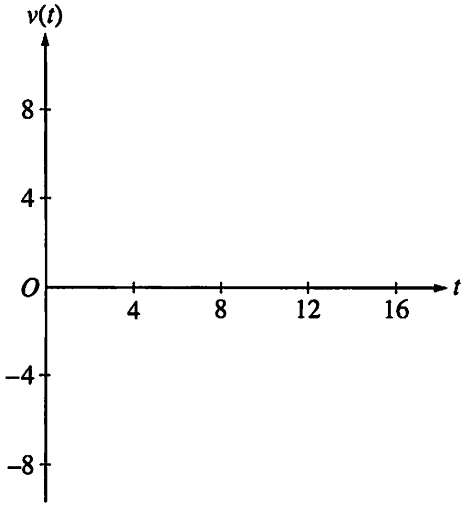
3. A particle moves along the x-axis so that its velocity v at any time t, for $0 \le t \le 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

   (a) On the axes provided, sketch the graph of $v(t)$ for $0 \le t \le 16$.

   (Note: Use the axes provided in the test booklet.)



   (b) During what intervals of time is the particle moving to the left? Give a reason for your answer.

   (c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.

   (d) Is there any time t, $0 < t \le 16$, at which the particle returns to the origin? Justify your answer.

END OF PART A OF SECTION II

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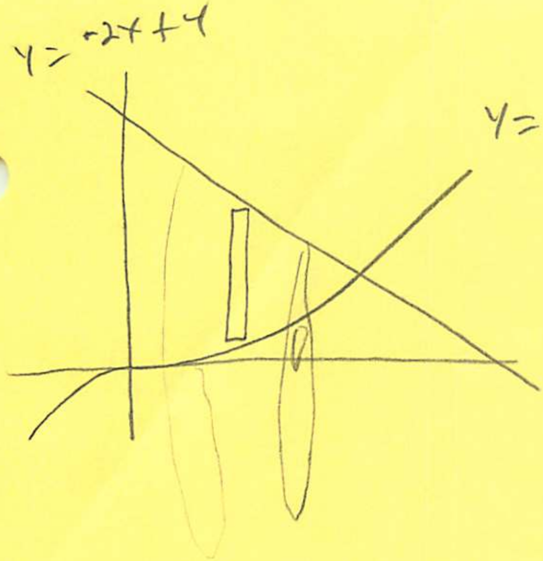
FORM B I



$y = \frac{x^3}{1+x^2}$

7 a) $\int_0^{1.488} -2x+4 - \left(\frac{x^3}{1+x^2}\right) dx =$

$Y_1 - Y_2 \overset{\text{set}}{\geq} 0$

3.215

4 b) $\pi \int_0^{1.488} Y_1^2 - Y_2^2 \, dx = 31.885$

4 c) $\int_0^{1.488} (Y_1 - Y_2)^2 \, dx = 8.997$

a) Proper integral 3 PTS Answer 1 PT Proper intersection 3 PTS

b) 3 PTS Answer 1 PT

c) 3 PTS Answer 1 PT

$P'(t) = 1 - 3e^{-.2\sqrt{t}}$ $P(0) = 50$

③ A) $P'(9) = -.646 < 0$ The level of pollutant is NOT increasing. at $t = 9$

③pts B) $P'(t) \overset{\text{set}}{=} 0$

sample = 5

$t = 30.174$

$P''(t) = .018 > 0 \therefore$ min at $t = 30.174$

f'' test.

$t = (5 \ln 3)^2$

⑤pts c) $\int P' = \int 1 - 3e^{-.2\sqrt{t}}\,dt$

$P(0) + \int_0^A 1 - 3e^{-.2\sqrt{t}}\,dt$

Pollutant at $t=0$ + Added (or subtracted) since $t=0$

$50 \quad -14.896 = 35.104$

Yes. It is safe because the pollutant is ≤ 40 gallons.

d) Tangent line to P at $t = 0$

⑤pts $(0, 50)$ $m = P'(0) = 1 - 3 = -2$

$y - 50 = -2(x - 0)$

$y = -2x + 50 \overset{\text{set}}{=} 40$

$2x = 10$

$x = 5$

The tangent line predicts a safe situation when $t = 5$.

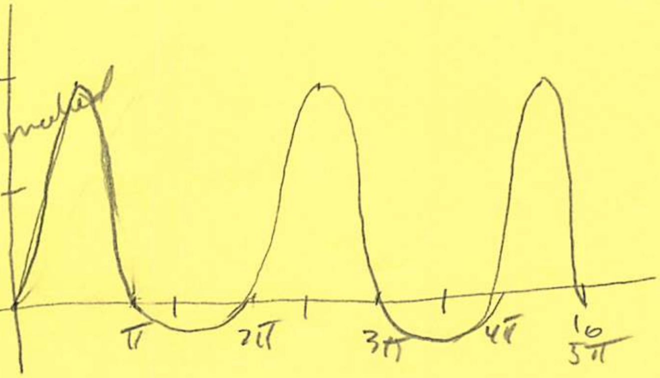
FORM B 3

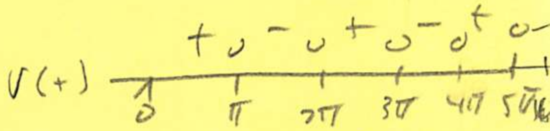
$v(t) = e^{2\sin t} - 1 \qquad x(0) = 0$

③ a)

3 PTS if π are marked

2 PTS otherwise





②

⑤ b) The particle moves left when $v(t) < 0$ ①

3 intervals

when $2\sin t = 0$

$t = k\pi$

1 PT per interval.

Particle moves left for $\pi < t < 2\pi$ and $3\pi < t < 4\pi$ and $5\pi < t < 16$

③ c) $\underbrace{\int_0^4 \text{abs}(e^{2\sin t} - 1)\,dt}_{\text{2 PTS}} = 10.542$

1 PT.

1/3 if no abs and they get 9.671

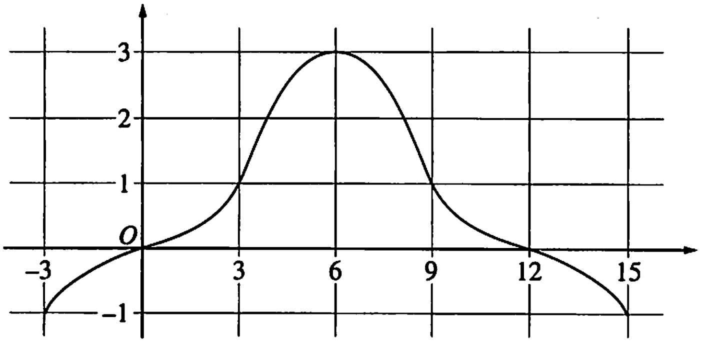
③ d) No the particle never returns to the origin.
the area under the curve between $0 < t < \pi$ is
much more than the "negative area" between
$\pi < t < 2\pi$. Thus these areas are the distance of
from the origin, the particle never returns,

NAME ______________

2002 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.

   (a) Find $g(6)$, $g'(6)$, and $g''(6)$.

   (b) On what intervals is g decreasing? Justify your answer.

   (c) On what intervals is the graph of g concave down? Justify your answer.

   (d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

---

5. Consider the differential equation $\frac{dy}{dx} = \frac{3 - x}{y}$.

   (a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

   (b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

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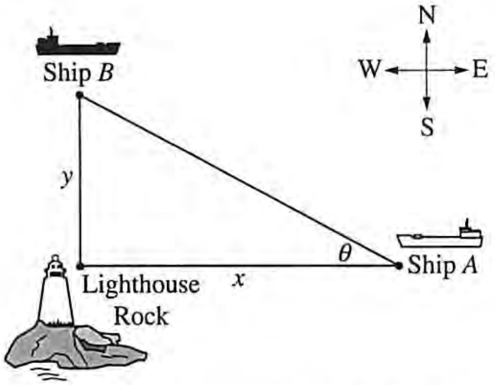
GO ON TO THE NEXT PAGE.



6. Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.

   (a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

   (b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

   (c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

END OF EXAMINATION

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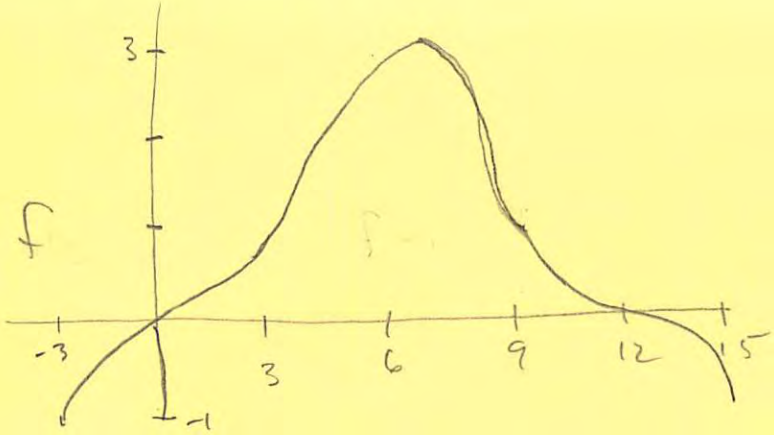
FORM B
2002
4



$g(x) = 5 + \int_6^x f(t)\,dt$

④ a) $g(6) = 5 + \int_6^6 f(t)dt = 5 + 0 = 5$ ①

$g'(6) = f(x)\Big|_{x=6} = 3$ ①

$g''(6) = f'(x)\Big|_{x=6} = 0$ ①

Given that f has horizontal tangent at $x=6$ ~~tangent~~ t f is 2nd der of g. ①

④ b) g decreases when $g' < 0$

reason 2pts

$g' = f(x)$ ∴

g decreases $-3 < x < 0$ and $12 < x < 15$

④ c) g concave down when $g'' < 0$. $g'' < 0$ when tangent to $f < 0$.

Reason 2PTS

∴ g concave down $6 < t < 15$

③ d) trapezoid approx.

$\frac{3}{2}\left[-1 + 2\cdot 0 + 2\cdot 1 + 2\cdot 3 + 2\cdot 1 + 2\cdot 0 - 1\right]$ ②

$\frac{3}{2}\left[-1 + 2 + 6 + 2 - 1\right]$

12 ①

form B
2002
5

$\frac{dy}{dx} = \frac{3-x}{y}$ $1 < x < 5$ $y = -2$

⑦ a)

$y\frac{dy}{dx} = 3 - x\, dx$

$\frac{1}{2}y^2 = 3x - \frac{x^2}{2} + C$

$\frac{dy}{dx} \overset{\text{set}}{=} 0 = \frac{3-x}{y}$

③ $x = 3$

$\frac{dy}{dx}$ $-$ 0 $+$ at $x=3$

$y = -2$

top $+$ 0 $-$ at 1, 3

bottom $-$

Reasoning ③

Sentences ① y is tangent at $x = 3$
this point is a min
by f' test.

⑧ b) $y = g(x)$ $-2 < x < 8$ $\overset{x}{g(6)} = \overset{y}{-4}$ find g(x)

$\frac{1}{2}y^2 = 3x - \frac{x^2}{2} + C$

$\frac{1}{2} \cdot 16 = 3 \cdot 6 - \frac{36}{2} + C$

$8 = 18 - 18 + C$

$\therefore C = 8$

③

$\frac{1}{2}y^2 = 3x - \frac{x^2}{2} + 8$

$y^2 = 6x - x^2 + 16$

③ $y = \pm\sqrt{16 + 6x - x^2}$

$(8 - x)(2 + x)$

$-$ 0 $+$ 0 $-$ at -2, 8; 0

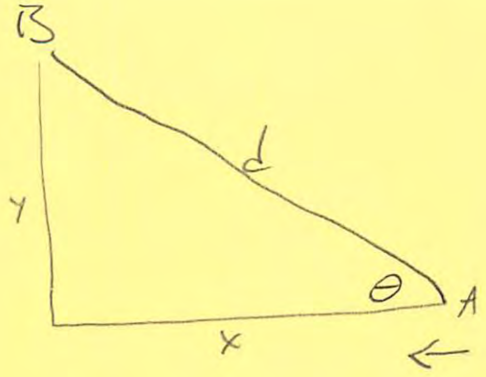
choose $y = -\sqrt{16 + 6x - x^2}$

so $g(x) = -4$

②

Form B
2002
6

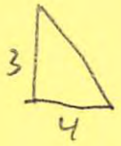
⑥



$\frac{dx}{dt} = -15$

$\frac{dy}{dt} = 10$

① a)



distance between ships $\sqrt{3^2+4^2} = 5$
The Ships are 5 Km apart.

⑥ b) $\left.\frac{dd}{dt}\right| = ?$ ②
$x = 4$
$y = 3$

$d^2 = x^2 + y^2$

$\cancel{2}d\frac{dd}{dt} = \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}$ ③

$\frac{dd}{dt} = \frac{4(-15) + 3 \cdot 10}{5}$ No $\frac{dx}{dt}$ $\frac{dy}{dt}$ 1/6

$-\frac{30}{5} = -6$

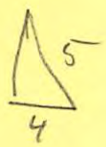
① distance between ships is shortening at 6 Km/hr.

they get 18 if $\frac{dx}{dt} = +15$ 4/6

8 c) $\frac{d\theta}{dt} = ?$ ①

② $\tan\theta = \frac{y}{x}$
Relationship

$\sec^2\theta \frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$ ③



$\frac{d\theta}{dt} = \frac{4 \cdot 10 - 3(-15)}{16} \cdot \cos^2\theta$

$\cos\theta = \frac{4}{5}$

$= \frac{85}{16}\cos^2\theta$

$\frac{85}{\cancel{16}} \cdot \frac{\cancel{16}}{25}$ ② Very cool!

$\frac{d\theta}{dt} = \frac{17}{5}$ radians/hr. ①
The angle is increasing at $\frac{17}{5}$ rad/hr

and
$\frac{d\theta}{dt} = -\frac{1}{5}$
if $\frac{dx}{dt} = +15$

2002

	Both Right	+
	one Right	✓
one Right other not Done	or Tried Both	0
	Tried one	–
	nothing	=

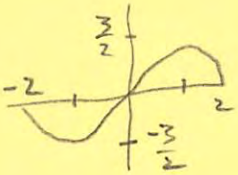
I c $\min h(x) \simeq -2.330$
$\max h(x) = e \simeq 2.718$

II c $H(17) = 3725$ means that at 5pm there are 3725 people in the park

$H'(17) = -380.281$ means that at 5pm the park population is decreasing at about 380 people per hour.

III B Both stmts are true. must supply reasons why they are true.

IV D Pict looks like sine curve



V B The water evaporates at $\frac{15\pi}{8}$ cm³/hr.

VI B $y = 5x - 9$

$y(1.2) = -3$

This is less than the actual value because f is concave up ($f''(x) > 0\ \forall x$) Therefore f is increasing faster than the constant increase of a straight line.

2002 Calculu8s AB Free Response Questions.

You may use a calculator on problems 1, 2, and 3 ONLY. Allow 45 minutes with a calculator and an additional 45 minutes for problems 4, 5 and 6 without a calculator. If you may go back to problems 1, 2, or 3 in the second 45 minute period but you may not use a calcuator at that time.

1) Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

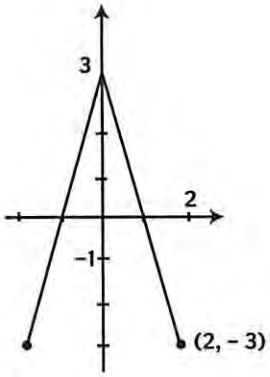
   a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.
   b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.
   c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$ and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

2) The rate at which people enter an amusement park on a given day is modeled by the function E defined by $E(t) = \dfrac{15600}{t^2 - 24t + 160}$. $L(t) = \dfrac{9890}{t^2 - 38t + 370}$ describes the rate at which people leave the same amusement park on the same day. Both $E(t)$ and $L(t)$ are measured in people per hour during which the park is open. The park opens at 9 am (t=9) and closes at 11 pm ($t = 23$). At time $t = 9$, there are no people in the park.

   a) How many people have entered the park by 5 pm ($t = 17$)? Round your answer to the nearest whole number.
   b) The price of admission to the park is \$15 until 5 pm ($t = 17$). After 5 pm, the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole dollar.
   c) Let $H(t) = \int_9^t E(x) - L(x)\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.
   d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

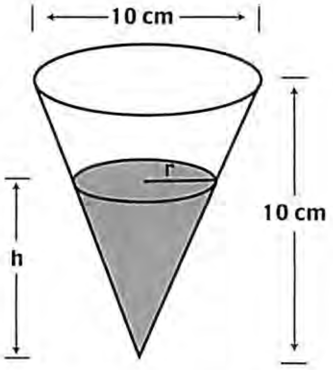
3) An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \ge 0$ is given by $v(t) = \sin\left(\frac{\pi}{3} t\right)$.

   a) What is the acceleration of the object at time $t = 4$?
   b) Consider the following two statements.
   Statement I: for 3 < t < 4.5, the velocity of the object is decreasing.
   Statement II: for 3 < t < 4.5, the speed of the object is increasing.
   Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.
   c) What is the total distance traveled by the object over the time interval $0 \le t \le 4$?
   d) What is the position of the object at time $t = 4$?



4. The graph of the function f shown consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

   a) Find $g(-1), g'(-1),$ and $g''(-1)$.
   b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.
   c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.
   d) Draw an axis and sketch the graph of g on the closed interval $[-2, 2]$.



5. A container has the shape of an open right circular cone, as shown. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr. ($V_{cone} = \frac{\pi}{3} r^2 h$)

   a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.
   b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.
   c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

6. Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$.
The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

a) Evaluate $\int_0^{1.5} 3f'(x) + 4\,dx$. Show the work that leads to your answer.

b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0 \end{cases}$.

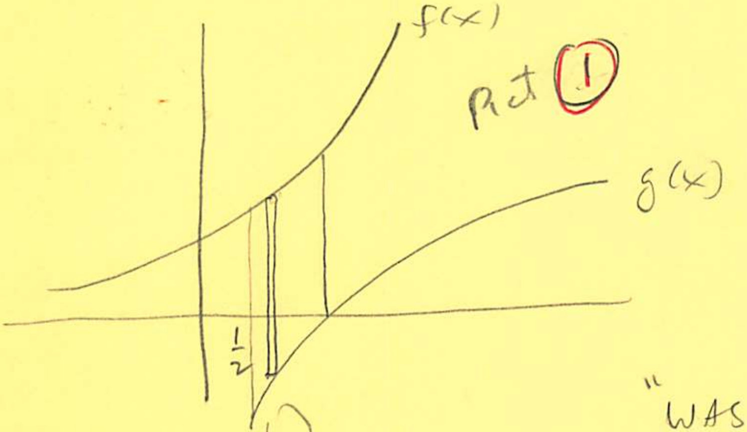
The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

I

2002

③

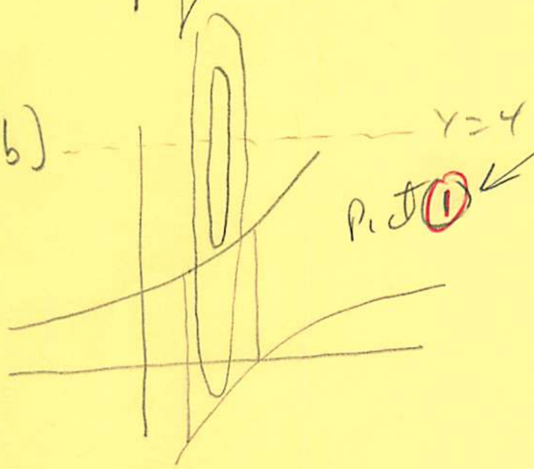
a) $f(x) = e^x \qquad g(x) = \ln x$



Pict ①

$$\int_{\frac{1}{2}}^{1} e^x - \ln x \, dx = 1.223$$ ① ①

③ b)



"Washers."

Pict ①

$$\pi \int_{\frac{1}{2}}^{1} (4 - \ln x)^2 - (4 - e^x)^2 \, dx$$ ①

23.609 ①

⑨ c) $h(x) = e^x - \ln x$ ①

$h'(x) = e^x - \frac{1}{x} \overset{\text{set}}{=} 0$

$xe^x - 1 = 0$

Solve using .7 (in interval)

$x = .567$ ①

$h(.567) = 2.330$ ①

$h''(x) = e^x + \frac{1}{x^2} > 0 \ \forall x \therefore$ min ① f'' test.

$h(.5) = 2.342$ ①

$h(1) = e$

h': $-$ 0 $+$ on $\overset{\uparrow}{0}$, $\frac{1}{2}$, $.567$, 1

$h'(1) = e > 0$

① This is only 1 min ∴ it is the absolute min.

① $x = .5$ is a max ENDPT dec from it

① $x = 1$ is a max ENDPT inc to it

① e is Abs max.

II
2002

Rate of entry
$$E(t) = \frac{15600}{t^2-24t+160}$$ in people/hr.

Rate of Leaving
$$L(t) = \frac{9890}{t^2-38t+370}$$

$9 \le t \le 23$

at t=9 No people in Park.

④ a) $\int_9^{17} E(t)\,dt$ ② $= 6004$ ①   6004 people have entered The park by 5 pm. ①

③ b) $\int_{17}^{23} E(t)\,dt = 1271$ ① # of late entry people.

6004 · 15 + 1271 · 11 ①

90064 + 13984

$104,048

Park collects ① $104,048 per day.

a total of 104041 would be acceptable

⑥ c) $H(t) = \int_9^t (E(x)-L(x))\,dx \quad 9 \le t \le 23$

$H(17) = 3725$ ①

$H'(17) = E(17) - L(17)$

$380.488 - 760.769$

$= -380.281$ ①

② H(17) = 3725 means that at 5pm there are 3725 People in the park.

② H'(17) = −380.281 means that at 5pm the park population is decreasing at about 380 people per hour.

II
2002

② d) H is # of people in park

$H' = E(t) - L(t) \overset{\text{set}}{=} 0$

$E(t) = L(t)$

H' + 0 −
9 15,795

① $t = 15.795$

$E(15) - L(15) = 228.4 > 0$

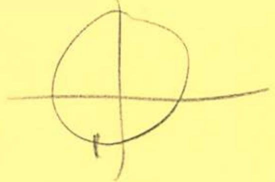
max population happens at $t = 15.795$ by max f' test. ①

which is just before 4 o'clock.

III

2002

$x(0)=2$ $\frac{dx}{dt}=v=\sin\frac{\pi}{3}t$



② a) acceleration is deriv of v. ①

$$a(t)=\frac{\pi}{3}\cos\frac{\pi}{3}t\Big|_{4}$$

$$\frac{\pi}{3}\cos\frac{4\pi}{3}=-\frac{\pi}{6}=-.524 \quad ①$$

the acceleration is −.524. this is deceleration of .524.

⑥ b) $\frac{\pi}{3}t\overset{\text{set}}{=}\frac{(2K+1)\pi}{2}$

$t=\frac{3}{2}(2K+1)$

$K=-1$ $t=-\frac{3}{2}$

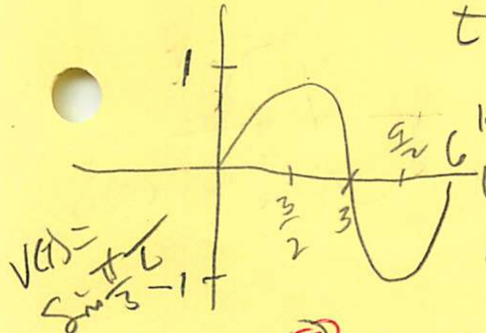
$K=0$ $t=\frac{3}{2}=1.5$

$K=1$ $t=\frac{9}{2}=4.5$

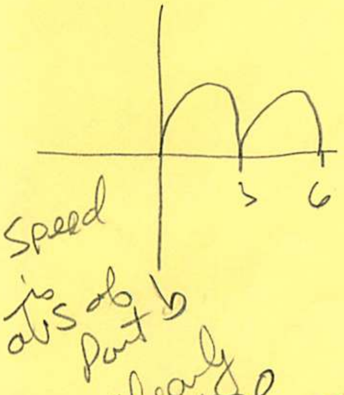
a(t) + 0 − 0 +

0 $T=\frac{3}{2}$ 1.5 3 $T=\frac{9}{2}$ 4.5

a(0) > 0



③ for the interval $3<t<4.5$ the velocity is decreasing because the acceleration is less than zero, so I is true.



Speed is abs of part b clearly speed ≥ 0

speed is abs (velocity) V(t) + 0 − 0 +

0 T=3 T=6

$\frac{\pi}{3}t\overset{\text{set}}{=}K\pi$

$t=3K$

$K=0$ $t=0$

$K=1$ $t=3$

$K=2$ $t=6$

T=1

$\sin\frac{\pi}{3}>0\ \therefore +$

ABS V + 0 + 0 +

0 3 6

③

$V(3)=0$

$V(4.5\cdot\frac{\pi}{3})<0$ ABS V( ) > 0 so "speed" is increasing in the interval. Statement II is also true.

III

2002

c) $\int_0^4 \left|\sin \frac{\pi t}{3}\right| dt = 2.387$

③ with abs or breaking up integral.

1 PT if ignore reversal.

④ d) $dx = \int \sin \frac{\pi}{3} t \, dt$

$$x = \frac{-3}{\pi} \cos \frac{\pi t}{3} + c$$

$$\underbrace{x(0) = 2}_{\text{Given}} \overset{\text{Set}}{=} -\frac{3}{\pi} \cos \frac{\pi}{2} \cdot 0 + c$$

$$= \frac{-3}{\pi} + c \quad ②$$

$$c = 2 + \frac{3}{\pi}$$

$$\therefore \; x(t) = \frac{-3}{\pi} \cos \frac{\pi t}{3} + 2 + \frac{3}{\pi}$$

$$x(4) = \frac{-3}{\pi} \cos \frac{4\pi}{3} + 2 + \frac{3}{\pi}$$

$$= 3.432 = \frac{3}{2\pi} + 2 + \frac{3}{\pi}$$

$$\frac{3 + 4\pi + 6}{2\pi}$$

② $\frac{4\pi + 9}{2\pi}$

OR Alternative:

$$x(4) = x(0) + \int_0^4 \sin \frac{\pi}{3} t \, dt$$

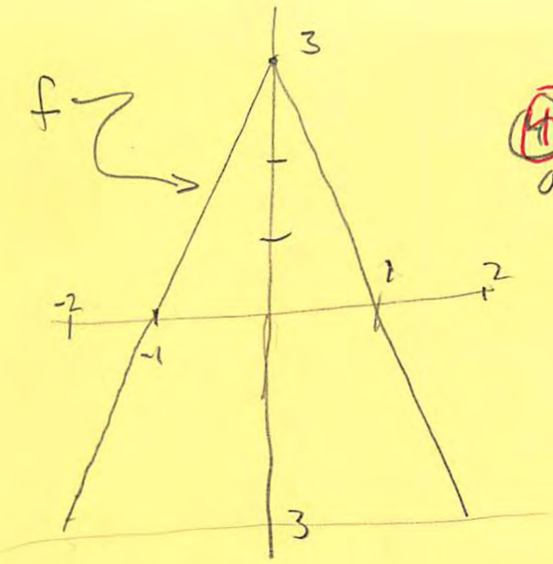
↑

This is the constant of integration.

④

IV

2002



④ a) $g(x)=\int_0^x f(t)\,dt$

$g(-1)=\int_0^{-1} f(t)\,dt = \frac{1}{2}\cdot 1\cdot 3 = \frac{-3}{2}$ ②

$g'(-1)= f(-1)=0.$ ①

$g''(-1)=$ Slope of line $(-1,0)$ $(0,3)$

$= \frac{3}{1} = 3$ ①

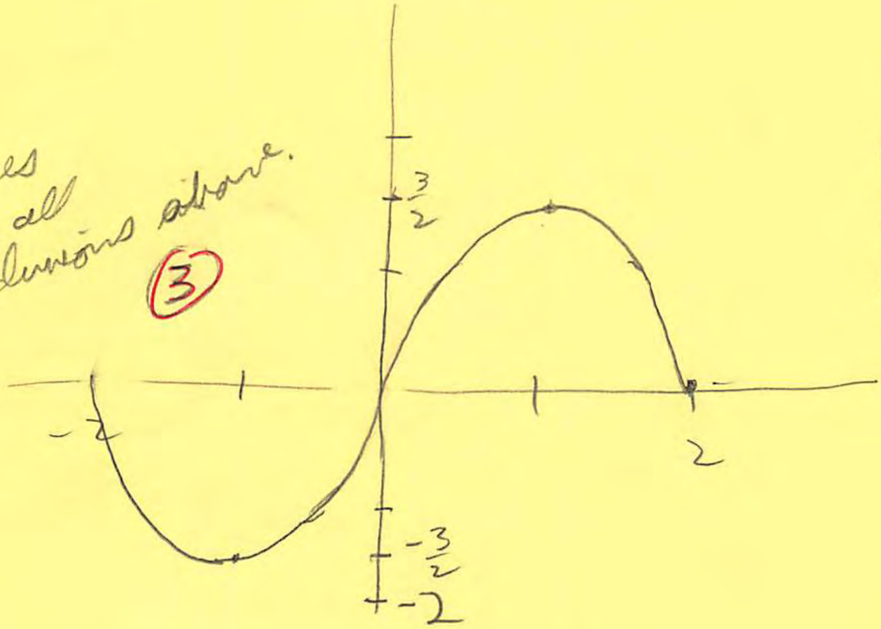
③ b) g is increasing when f is >0 ② and f>0 when $-1<x<1$ ①

③ c) g is concave down when $f'<0$ ② and $f'<0$ when f is decreasing.

g is concave down $0<x<2$ ①

⑤ d)

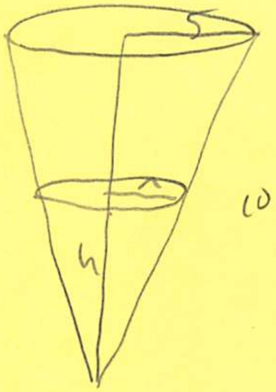
$g(0)=0$ ←
$g(-1)=\frac{-3}{2}$
$g(1)=\frac{3}{2}$ ←
$g(2)=0$ ←
$g(-2)=0$ ←
②



Pict agrees with all conditions above. ③

$\overline{\underline{V}}$

2002



$$\frac{dh}{dt} = -\frac{3}{10} \qquad V = \frac{\pi}{3} r^2 h$$

$$\frac{r}{h} = \frac{5}{10}$$

$$r = \frac{1}{2} h \longrightarrow V(h) = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h$$

$$V = \frac{\pi}{12} h^3$$

eliminate $\overset{\wedge}{}$ r ③ style

⑥ a) $V(5) = \frac{\pi}{12} \cdot 5^3$ ②

$= \frac{125\pi}{12}$ cm^3 there are ^ of water, when $h = 5$ ①

⑤ b) $\left.\frac{dV}{dt}\right|_{h=5} = ?$ ①

$$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$= \frac{\pi}{4} \cdot \cancel{25}^{5} \left(\frac{-3}{\cancel{10}_{2}}\right)$$ ①

③ $= -\frac{15\pi}{8}$ cm^3 per hr.

The water evaporates at $\frac{15\pi}{8}$ cm^3 per hr.

④ c) WTS $\frac{dV}{dt} = K \cdot \pi r^2 = K\pi\left(\frac{h^2}{4}\right)$

↑ from Part "a"

from Part b → $\frac{\cancel{\pi}}{\cancel{4}} \cancel{h^2} \frac{dh}{dt} = \frac{K \cancel{\pi} \cancel{h^2}}{\cancel{4}}$

$\frac{dh}{dt} = K \therefore K = \frac{-3}{10}$ ③

The proportionality constant is $\frac{-3}{10}$

①

$\underline{VI}$
2002

③ a) $\int_0^{1.5} 3f'(x)+4\,dx \overset{FTC}{=}$

$3f(x)+4x\Big|_0^{1.5}$ ②

$3f(1.5)+4(1.5)-(3f(0)+4(0))$

$3(-1)+6-(3\cdot(-7)+0)$

$-3+6+21$

24 ①

④ b) $(1, f(1))\quad m=f'(1)$

$(1,-4)\quad m=5.$

$y+4=5(x-1)$

$y+4=5x-5$

$y=5x-9$ ①

$y(1.2)=5(1.2)-9$

$=6-9$

$=-3$ ①

The approximate value of $f(1.2)$ is -3.
This estimate is less than the actual value because f is concave up
$(f''(x)>0\ \forall x)$

② Therefore f is increasing faster than the constant increase of a straight line.

ie any tangent is below the graph because $f''(x)>0$

③ c)

$\underline{VI}$
2002

$0 < c < .5 \qquad f''(c) = r$

f''(c) is slope of f' $\quad \dfrac{f'(.5) - f'(0)}{.5 - 0} \overset{set}{=} f''(c)$

by mean value thm ②

"must" requires mention of MVT.

$$\frac{3 - 0}{\frac{1}{2}} = 6 = r$$ ①

one value which must exist for r is 6 as a result of the mean value thm.

D) ⑤

consider ....

$$g(x) = \begin{cases} 2x^2 - x - 7 & x < 0 \\ 2x^2 + x - 7 & x \geq 0 \end{cases}$$

$g''(x) = 4$
but in part c
$f''(c) = 6$
so $f \neq g$.

$$g'(x) = \begin{cases} 4x - 1 & x < 0 \\ 4x + 1 & x > 0 \end{cases}$$

$g'(0) = -1$

	-1.5	-1	-.5	0	.5	1	1.5
g							
g'				DNE			

$$\lim_{x \to 0^-} \frac{2x^2 - x - 7 + 7}{x}$$

$$\lim_{x \to 0^-} \frac{2x^2 - x}{x} \qquad \lim_{x \to 0^+} \frac{2x^2 + x - 7 + 7}{x}$$

$$\lim_{x \to 0^-} 2x - 1 \qquad \lim_{x \to 0^+} 2x + 1$$

$$-1 \neq +1$$

∴ g'(x) DNE

but $f'(0) = 0$ i.e. it does exist.

∴ $f \neq g$ They could differ by a constant! but not this der.

CALCULUS AB
SECTION II, Part A

Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

---

1. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

   (a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

   (b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

   (c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

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2. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round your answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

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3. An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

   (a) What is the acceleration of the object at time $t = 4$?

   (b) Consider the following two statements.

   Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.
   Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

   Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

   (c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

   (d) What is the position of the object at time $t = 4$?

END OF PART A OF SECTION II

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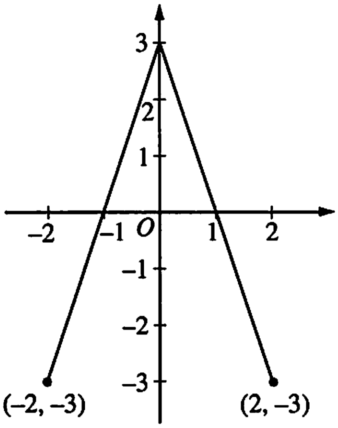
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

   (a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

   (b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

   (c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

   (d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.
   (Note: The axes are provided in the pink test booklet only.)

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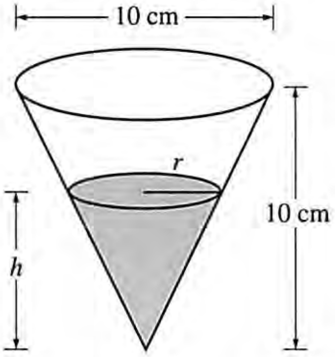
GO ON TO THE NEXT PAGE.



5. A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

(Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

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x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

6. Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} (3f'(x) + 4)\,dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0. \end{cases}$

The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

END OF EXAMINATION

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Solutions to the 2002 AP Calculus AB Exam

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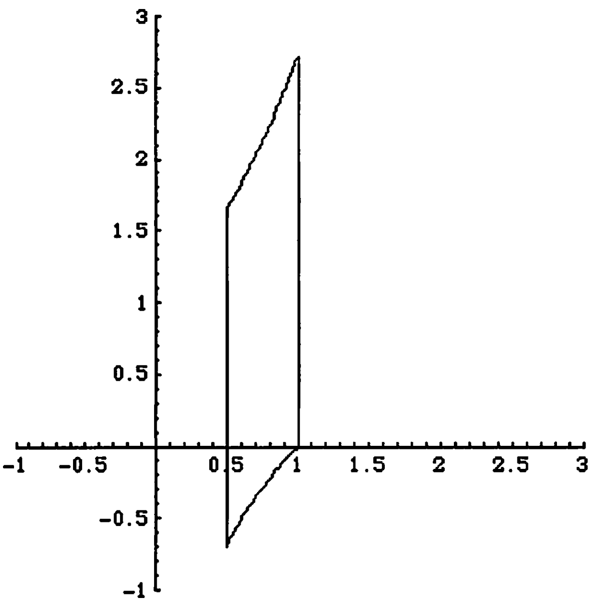
Metropolitan State College of Denver

1.

a.

Here is the region between the graphs on the specified interval:

```
p1 = Plot[{ⅇ^x, Log[x]}, {x, 1/2, 1},
  PlotRange → {{-1, 3}, {-1, 3}},
  AspectRatio → Automatic,
  DisplayFunction → Identity];
p2 = Graphics[
  {Line[{{1/2, ⅇ^(1/2)}, {1/2, Log[1/2]}}],
   Line[{{1, ⅇ}, {1, 0}}]}];
Show[p1, p2,
 DisplayFunction → $DisplayFunction]
```



```
- Graphics -
```

The area is given by

$$\int_{1/2}^{1} (e^{x} - \mathrm{Log}[x])\,dx$$

$$\frac{1}{2}\left(1-2\sqrt{e}+2e-\text{Log}[2]\right)$$

And here, for the curious, is a numeric answer:

```
N[%]
```

```
1.22299
```

b

Using the method of washers, we find that the required volume is

$$\pi\int_{1/2}^{1}\left((4-\text{Log}[x])^2-(4-e^x)^2\right)dx$$

$$\frac{1}{2}\pi\left(10-16\sqrt{e}+17e-e^2-10\,\text{Log}[2]-\text{Log}[2]^2\right)$$

(Use integration by parts twice in succession to find $\int(\ln x)^2\,dx$.) Numerically, this is

```
N[%]
```

```
23.6095
```

c

Let

```
h[x_] = e^x - Log[x]
```

$e^x-\text{Log}[x]$

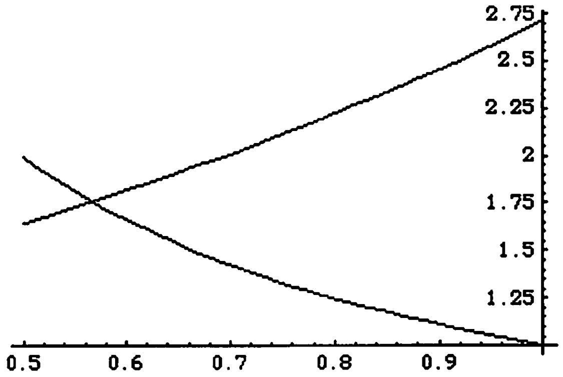
We seek the critical pointsof h that lie in the interval $\left(\frac{1}{2},1\right)$ and points in the same interval where the derivative is undefined.

```
h'[x]
```

$$e^x-\frac{1}{x}$$

This is defined everywhere in $\left(\frac{1}{2},1\right)$, so we consider only critical points. Examination of a graph

$$\text{Plot}\left[\left\{e^x,\frac{1}{x}\right\},\left\{x,\frac{1}{2},1\right\}\right]$$



```
- Graphics -
```

indicates that there is exactly one critical point. We find it numerically:

```
b = x /. FindRoot[h'[x] == 0, {x, 0.75}]
```

```
0.567143
```

The points of interest to us are the endpoints and the critical point, because both the absolute maximum and the absolute minimum must occur at such points. Thus we evaluate h at each of the points 0.5, 0.567143, and 1.0.

```
Map[h, {0.5, b, 1.0}]
```

```
{2.34187, 2.33037, 2.71828}
```

The largest of these outputs is the number e itself, so it is the maximum value taken on by h on $\left[\frac{1}{2}, 1\right]$. The smallest output is 2.33037, so it is the minimum value taken on by h on $\left[\frac{1}{2}, 1\right]$.

2.

a.

We are given

$$\mathtt{EnterRate[t\_]} = \frac{15600}{t^2 - 24\,t + 160}$$

$$\frac{15600}{160 - 24\,t + t^2}$$

$$\mathtt{LeaveRate[t\_]} = \frac{9890}{t^2 - 38\,t + 370}$$

$$\frac{9890}{370 - 38\,t + t^2}$$

From these and the fact that there are no people in the park at 9:00, we have

$$\text{NumberEntered}[t\_] = \int_9^t \text{EnterRate}[\tau]\,d\tau$$

$$15600\left(\frac{1}{4}\,\text{ArcTan}\left[\frac{3}{4}\right] + \frac{1}{4}\,\text{ArcTan}\left[\frac{1}{4}\,(-12+t)\right]\right)$$

Consequently, at 5:00 pm,

```
NumberEntered[17.0]
```

```
6004.27
```

so that 6004 people have entered the park by 5:00 pm.

b.

Revenue is given by

```
15 NumberEntered[17] +
 11
  (NumberEntered[23] - NumberEntered[17])
```

$$234000\left(\frac{1}{4}\,\text{ArcTan}\left[\frac{3}{4}\right] + \frac{1}{4}\,\text{ArcTan}\left[\frac{5}{4}\right]\right) + \\ 11\left(-15600\left(\frac{1}{4}\,\text{ArcTan}\left[\frac{3}{4}\right] + \frac{1}{4}\,\text{ArcTan}\left[\frac{5}{4}\right]\right) + \\ 15600\left(\frac{1}{4}\,\text{ArcTan}\left[\frac{3}{4}\right] + \frac{1}{4}\,\text{ArcTan}\left[\frac{11}{4}\right]\right)\right)$$

```
N[%] // InputForm
```

```
104048.16522947158
```

Rounded to the nearest dollar, as required, this gives $104,048.

c.

If $H[t] = \int_9^t (\text{EnterRate}[x] - \text{LeaveRate}[x])\,dx$, then $H'[t] = \text{EnterRate}[t] - \text{LeaveRate}[t]$. (This is just the Fundamental Theorem of Calculus.) Hence $H'[17]$ is given by

```
EnterRate[17] - LeaveRate[17]
```

$$-\frac{202690}{533}$$

or, numerically,

```
N[%]
```

```
-380.281
```

gives the number of people in the park at time , where . Thus, is the number of people in the park at 5:00 pm. $H'[t]$ gives the rate at which the number of people in the park is increasing at time t, again for $9 \le t \le 23$. $H'[17] = -380$ means that at 5:00 pm people are leaving the park at a rate of 380 people per hour.

d.

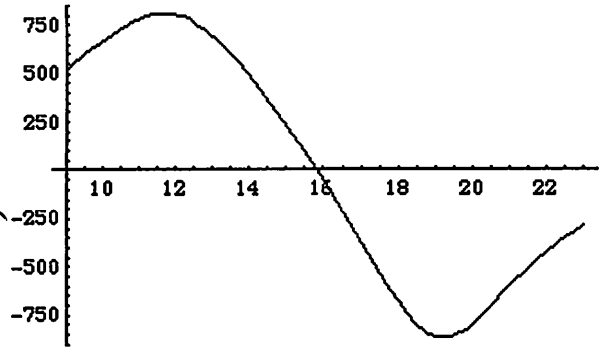
We have an expression for $H'[t]$:

```
HPrime[t_] =
 EnterRate[t] - LeaveRate[t] //
  Together
```

$$\frac{10\,(418960 - 35544\,t + 571\,t^2)}{(370 - 38\,t + t^2)\,(160 - 24\,t + t^2)}$$

Let's look at a picture to see if surprises are likely:

```
Plot[HPrime[t], {t, 9, 23}]
```



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- Graphics -
```

Looks pretty simple; the maximum should occur around 4 o'clock in the afternoon. Let's nail it:

```
Solve[HPrime[t] == 0, t]
```

$$\left\{\left\{t \to \frac{4}{571}\left(4443 - \sqrt{4788614}\right)\right\},\ \left\{t \to \frac{4}{571}\left(4443 + \sqrt{4788614}\right)\right\}\right\}$$

```
N[%]
```

```
{{t→ 15.7948}, {t→ 46.4539}}
```

The second root is outside the region of interest, but the first one is right where we thought it should be. The model therefore predicts that the number of people in the park is a maximum when $t = \frac{4}{571}\left(4443 - \sqrt{4788614}\right)$.

3.

a.

We are given

$$v[t\_] = \text{Sin}\left[\frac{\pi}{3}\, t\right]$$

$$\text{Sin}\left[\frac{\pi t}{3}\right]$$

$$x_0 = 2$$

$$2$$

Acceleration is $v'[t]$:

$$v'[t]$$

$$\frac{1}{3}\pi\,\text{Cos}\left[\frac{\pi t}{3}\right]$$

$$\% \;/.\; t \to 4$$

$$-\frac{\pi}{6}$$

Acceleration at $t = 4$ is $\frac{-\pi}{6}$. (The problem gives no units, so we give none.)

b.

Note that if $3 < t < 4.5$, then $\pi < \frac{\pi t}{3} < \frac{3\pi}{2}$, and we know that the cosine function is negative on $\left(\pi, \frac{3\pi}{2}\right)$. Because $v'[t]$ is a positive constant times $\cos\left(\frac{\pi t}{3}\right)$, we conclude that $v'[t]$ is negative on $(3, 4.5)$. This means that $v[t]$ is decreasing on $(3, 4.5)$. Thus, Statement I is correct.

On the other hand, the sine function takes on negative values in the interval $\left(\pi, \frac{3\pi}{2}\right)$, so if $s[t]$ denotes speed, then $s[t] = -\sin\left(\frac{\pi}{3}\, t\right) = -v[t]$ for $3 < t < 4.5$. Thus, $s'[t] = -v'[t]$ on that interval, and so $s'[t] > 0$ on $(3, 4.5)$. This means that speed increases throughout $(3, 4.5)$. Statement II is also correct.

c.

Total distance is the integral of speed, or $\int_0^4 s[\tau]\,d\tau$, in this case. Because $v[t] = \sin\left(\frac{\pi}{3}\, t\right)$ undergoes a sign-change at $t = 3$, we have total distance given by:

$$\int_0^3 v[\tau]\, d\tau - \int_3^4 v[\tau]\, d\tau$$

$$\frac{15}{2\pi}$$

Numerically, this is

```
N[X]
```

```
2.38732
```

d.

Letting $x[t]$ denote position at time t, we have, by the Fundamental Theorem of Calculus, $x[t] = x[0] + \int_0^t v[\tau]\,d\tau$. We were given that $x[0] = x_0 = 2$. Thus, position at time t is

$$2 + \int_0^t v[\tau]\,d\tau$$

$$2 + \frac{3}{\pi} - \frac{3\,\mathrm{Cos}\left[\frac{\pi t}{3}\right]}{\pi}$$

Setting t equal to 4 in this expression yields

```
X /. t → 4
```

$$2 + \frac{9}{2\pi}$$

Numerically, this is

```
N[X]
```

```
3.43239
```

?

4D

4.

a.

The function f, given pictorially, can be written as $f[x] = -3 + 3(x+2) = 3x + 3$ for $-2 \le x \le 0$; $f[x] = -3x + 3$ for $0 \le x \le 2$. Then $g[x] = \int_0^x (3t+3)\,dt = 3x + \frac{3}{2}x^2$ when $-2 \le x \le 0$, and $g[x] = \int_0^x (-3t+3)\,dt = 3x - \frac{3}{2}x^2$ when $0 < x < 2$. Thus, $g[-1] = -3 + \frac{3}{2} = \frac{-3}{2}$. (We could also have found this by looking at the area of the appropriate triangle in the figure.) By the Fundamental Theorem of Calculus, $g'[-1] = f[-1] = 0$. Also by the Fundamental Theorem of Calculus, $g''[-1] = f'[-1] = 3$.

b.

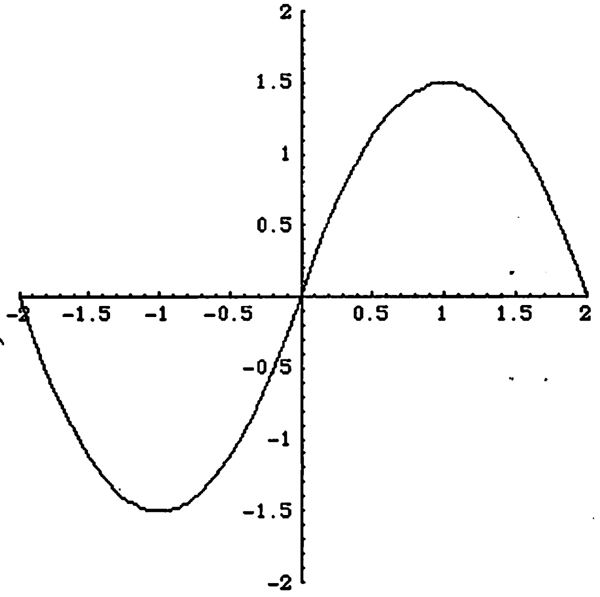
As we noted above for the special case $t = -1$, the Fundamental Theorem of Calculus tells us that $g'[t] = f[t]$. Thus, g is increasing on the closures of those intervals where $f[t] > 0$. From the picture, we conclude that g is an increasing function on the interval $[-1, 1]$, and on that interval only.

c.

From , which we derived above, we conclude that . Now for , and for $0 < t < 2$. We conclude that g is concave downward on the interval $(0, 2)$, where $f'[t] < 0$.

d.

```
p1 = Plot[3 x + 3/2 x^2, {x, -2, 0},
   DisplayFunction → Identity];
p2 = Plot[3 x - 3/2 x^2, {x, 0, 2},
   DisplayFunction → Identity];
Show[p1, p2,
  PlotRange → {{-2, 2}, {-2, 2}},
  AspectRatio → Automatic,
  DisplayFunction → $DisplayFunction]
```



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- Graphics -
```

5.

a.

When $h = 5$, we have $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{5}{2}\right)^2 5 = \frac{125\pi}{12}$ cm^3. (By similar triangles, $r = \frac{h}{2}$.)

b.

For all t we have $V[t] = \frac{1}{3}\pi r[t]^2 h[t] = \frac{1}{12}\pi h[t]^3$. Thus $V'[t] = \frac{1}{4}\pi h[t]^2 h'[t] = -\frac{3}{40}\pi h[t]^2$. This gives $V'[5] = -\frac{15}{8}\pi\frac{\text{cm}^3}{\text{hr}}$.

c.

As we noted above, $h[t] = 2 r[t]$, by similar triangles. Thus, $V'[t] = -\frac{3}{40}\pi h[t]^2 = -\frac{3}{10}\pi r[t]^2$. But $A[t] = \pi r[t]^2$. Thus,

$V'[t] = -\frac{3}{10} A[t]$, so that $V'[t]$ is proportional to $A[t]$ with constant of proportionality $-\frac{3}{10}$.

6.

a.

By the Fundamental Theorem of Calculus, $\int_0^{1.5} (3 f'[x] + 4)\, dx = (3 f[x] + 4x) |_0^{1.5} = (3 f[1.5] + 4(1.5)) - (3 f[0] + 4(0)) = (-3 + 6) - (-21 + 0)$. Thus, the value of the integral is 24.

b.

The equation of the tangent line at (x_0, y_0) is $y = y_0 + f'[x_0](x - x_0)$, so the required line has equation $y = -4 + 5(x - 1)$, or $y = 5x - 9$. We obtain an approximate value for $f[1.2]$ by substituting $x = 1.2$ into the equation for the tangent line: $f[1.2] \sim 5(1.2) - 9 = -3$. We have been given that $f''[x] > 0$ everywhere in the region of interest, so we know that the curve is concave upward in that region. This means that the tangent line at any point in the region lies below the curve throughout the region. Consequently, our estimate of -3 for $f[1.2]$ is an underestimate: $f[1.2] > -3$.

c.

We are given that $f''[x]$ exists throughout $[-1.5, 1.5]$. Consquently, f' is continuous throughout $[-1.5, 1.5]$. By the Mean Value Theorem, there must be a number c strictly between 0 and 0.5 such that $f''[c](0.5 - 0) = f'[0.5] - f'[0]$, Hence $f''[c] = (f'[0.5] - f'[0])/0.5 = 6$. The required number r is therefore 6.

d.

It is not possible that f and g are the same. As we have seen in part c. above, f' must be continuous throughout $[-1.5, 1.5]$ because we have been given the existence of $f''[x]$ in that interval. However, $g'[x] = 4x - 1$ when $x < 0$, and $g'[x] = 4x + 1$ when $x > 0$. It follows that g' is not continuous at $x = 0$.

Note: It also follows from these observations that $g'[0]$ is not defined. Derivatives have the Intermediate Value Property, and so cannot have jump discontinuities. This is not on the AB syllabus; it is, however, possible to see directly that $g'[0]$ is undefined.

Converted by *Mathematica* May 10, 2002

CALCULUS BC
SECTION II, Part A

Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

---

1. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = \ln x$.

   (a) Find the area of the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$.

   (b) Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and $x = 1$ is revolved about the line $y = 4$.

   (c) Let h be the function given by $h(x) = f(x) - g(x)$. Find the absolute minimum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of $h(x)$ on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

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GO ON TO THE NEXT PAGE.

2. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions arc valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round your answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$, and explain the meaning of $H(17)$ and $H'(17)$ in the context of the amusement park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

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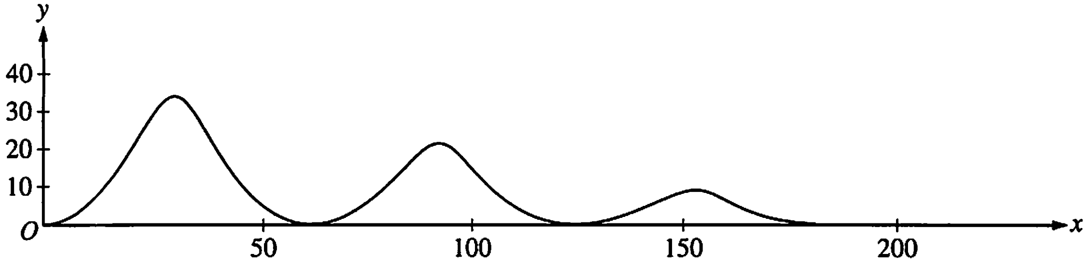

3. The figure above shows the path traveled by a roller coaster car over the time interval $0 \le t \le 18$ seconds. The position of the car at time t seconds can be modeled parametrically by

$$x(t) = 10t + 4\sin t$$
$$y(t) = (20 - t)(1 - \cos t),$$

where x and y are measured in meters. The derivatives of these functions are given by

$$x'(t) = 10 + 4\cos t$$
$$y'(t) = (20 - t)\sin t + \cos t - 1.$$

(a) Find the slope of the path at time $t = 2$. Show the computations that lead to your answer.

(b) Find the acceleration vector of the car at the time when the car's horizontal position is $x = 140$.

(c) Find the time t at which the car is at its maximum height, and find the speed, in m/sec, of the car at this time.

(d) For $0 < t < 18$, there are two times at which the car is at ground level ($y = 0$). Find these two times and write an expression that gives the average speed, in m/sec, of the car between these two times. Do not evaluate the expression.

END OF PART A OF SECTION II

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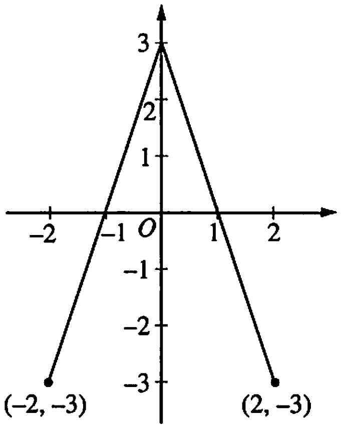
CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

   (a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

   (b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

   (c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

   (d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.
   (Note: The axes are provided in the pink test booklet only.)

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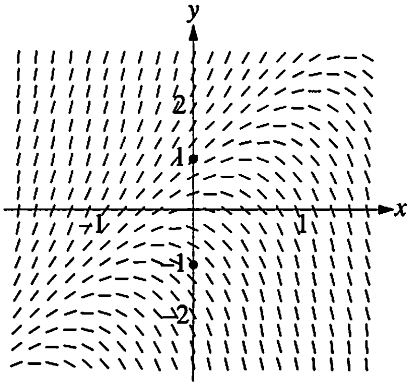
GO ON TO THE NEXT PAGE.

5. Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.

   (a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point $(0, 1)$ and sketch the solution curve that passes through the point $(0, -1)$.
   (Note: Use the slope field provided in the pink test booklet.)



   (b) Let f be the function that satisfies the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

   (c) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

   (d) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point $(0, 0)$? If so, is the point a local maximum or a local minimum? Justify your answer.

---

6. The Maclaurin series for the function f is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1} = 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4} + \cdots + \frac{(2x)^{n+1}}{n+1} + \cdots$$

   on its interval of convergence.

   (a) Find the interval of convergence of the Maclaurin series for f. Justify your answer.

   (b) Find the first four terms and the general term for the Maclaurin series for $f'(x)$.

   (c) Use the Maclaurin series you found in part (b) to find the value of $f'\left(-\frac{1}{3}\right)$.

END OF EXAMINATION

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AP® Calculus BC
2002 Free-Response Questions
Form B

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2002 AP® CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

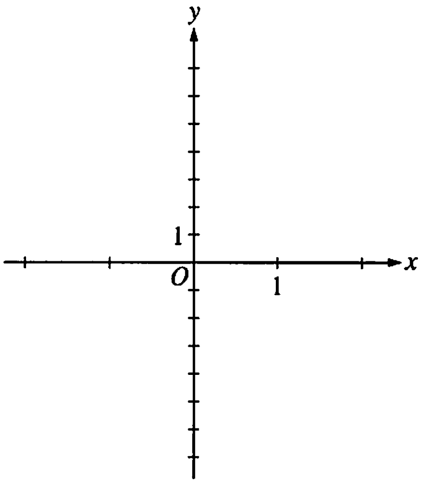
CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

---

1. A particle moves in the xy-plane so that its position at any time t, for $-\pi \le t \le \pi$, is given by $x(t) = \sin(3t)$ and $y(t) = 2t$.

   (a) Sketch the path of the particle in the xy-plane provided. Indicate the direction of motion along the path. **(Note: Use the axes provided in the test booklet.)**



   (b) Find the range of $x(t)$ and the range of $y(t)$.

   (c) Find the smallest positive value of t for which the x-coordinate of the particle is a local maximum. What is the speed of the particle at this time?

   (d) Is the distance traveled by the particle from $t = -\pi$ to $t = \pi$ greater than 5π ? Justify your answer.

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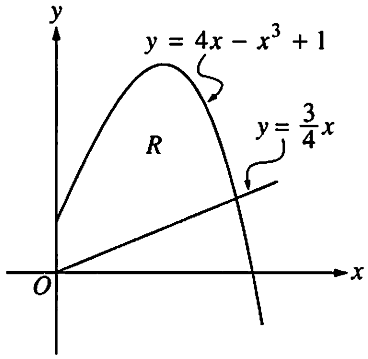
2. The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

   (a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

   (b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

   (c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

   (d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?



3. Let R be the region in the first quadrant bounded by the y-axis and the graphs of $y = 4x - x^3 + 1$ and $y = \frac{3}{4}x$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) Write an expression involving one or more integrals that gives the perimeter of R. Do not evaluate.

END OF PART A OF SECTION II

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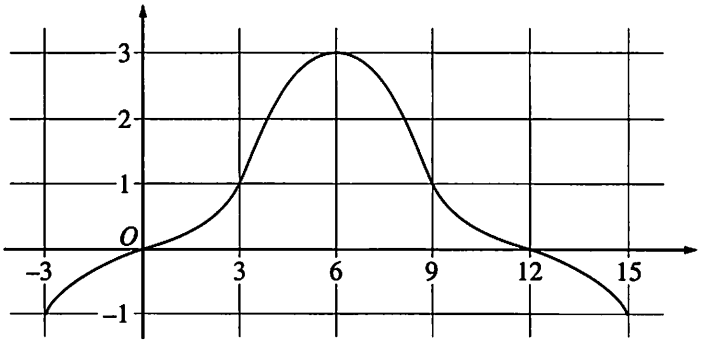
CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.

   (a) Find $g(6)$, $g'(6)$, and $g''(6)$.

   (b) On what intervals is g decreasing? Justify your answer.

   (c) On what intervals is the graph of g concave down? Justify your answer.

   (d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

---

5. Consider the differential equation $\frac{dy}{dx} = \frac{3 - x}{y}$.

   (a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

   (b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

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6. The Maclaurin series for $\ln\left(\frac{1}{1-x}\right)$ is $\sum_{n=1}^{\infty} \frac{x^n}{n}$ with interval of convergence $-1 \le x < 1$.

   (a) Find the Maclaurin series for $\ln\left(\frac{1}{1+3x}\right)$ and determine the interval of convergence.

   (b) Find the value of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

   (c) Give a value of p such that $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges, but $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ diverges. Give reasons why your value of p is correct.

   (d) Give a value of p such that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges, but $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ converges. Give reasons why your value of p is correct.

END OF EXAMINATION

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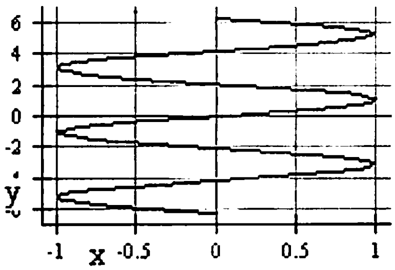
BC1



a) ↑ is direction of motion along path.

List of values where x = 0 from bottom to top: $-2\pi, -\frac{4\pi}{3}, -\frac{2\pi}{3}, 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$

b) $\boxed{\text{Range of } x(t) = -1 \le x \le 1 \quad \text{Range of } y(t) = -2\pi \le y \le 2\pi}$

c) $x'(t) = 3\cos(3t)$ Set = 0 --> $3\cos(3t) = 0$ --> $3t = -\frac{\pi}{2}$ --> $t = -\frac{\pi}{6}$ or

$3t = -\frac{3\pi}{2}$ --> $t = -\frac{\pi}{2}$ $\boxed{t = -\frac{\pi}{2} \text{ is the local maximum}}$. The speed of the particle

is $\left|\vec{v}\right| = \sqrt{\left(x'(-\frac{\pi}{2})\right)^2 + \left(y'(-\frac{\pi}{2})^2\right)} = \boxed{2}$

d) $5\pi \sim 15.708$ Use arc length $\int_{-\pi}^{\pi} \sqrt{x'(t)^2 + y'(t)^2}\, dt = 17.973$

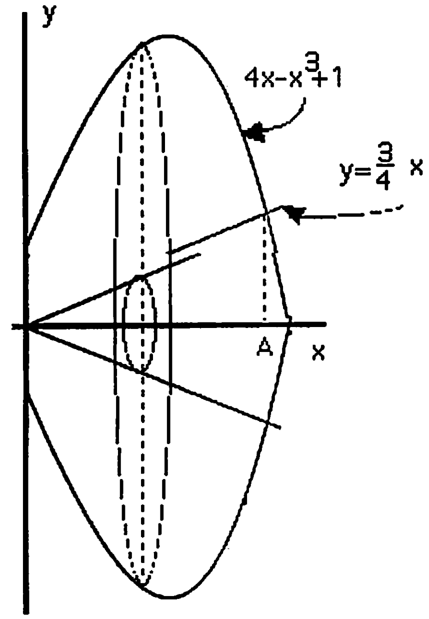
$\boxed{\text{The distance traveled is greater than } 5\pi}$

BC3

Note: Point of intersection of two curves is (1.940,1.455) Let A = 1.940

$y_1 = 4x - x^3 + 1 \qquad y_2 = \frac{3}{4}x$

a) $y = 4x - x^3 + 1$ is the upper curve, so $\int_0^A \left(4x - x^3 + 1 - \frac{3}{4}x\right) dx = \boxed{4.515}$



b) $\pi \int^{A} \left(\left(4x - x^3 + 1\right)^2 - \left(\frac{3}{4}x\right)^2\right) dx = \boxed{57.463}$

c) $\boxed{\int_0^A \sqrt{1+\left(4-3x^2\right)^2}\,dx + \int_0^A \sqrt{1+\left(\frac{3}{4}\right)^2}\,dx}$

BC6

Note: $\sum_{n=1}^{\infty} \frac{x^n}{n} = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots$

a) $\ln\left(\frac{1}{1+3x}\right) = \ln\left(\frac{1}{1-(-3x)}\right) =$

$= -3x + \frac{9x^2}{2} - \frac{27x^3}{3} + \frac{81x^2}{4} - \frac{243x^2}{5} + \ldots \quad \sum_{n=1}^{\infty} \frac{(-3x)^n}{n}$

$\lim_{n \to \infty} \left|\frac{(-3x)^{n+1}}{n+1} \cdot \frac{n}{(-3x)^n}\right| = \lim_{n \to \infty} \left(\frac{n}{n+1}\right) |-3x| = |3x|$

For $|3x| < 1 \to -1 < 3x < 1 \to -\frac{1}{3} < x < \frac{1}{3}$

$x = -\frac{1}{3} \to \sum_{n=1}^{\infty} \frac{1}{n}$ diverges and $x = \frac{1}{3} \to \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges, so

interval of convergence is $\boxed{-\frac{1}{3} < x \le \frac{1}{3}}$

b) Using $\ln\left(\frac{1}{1-x}\right) = \sum_{n=1}^{\infty} \frac{x^n}{n} = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots$ for $-1 \le x < 1$,

use $x = -1$ So $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = \ln\left(\frac{1}{1-(-1)}\right) = \boxed{\ln\left(\frac{1}{2}\right)}$

c) $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ diverges if $2p \le 1 \to p \le \frac{1}{2}$ $\quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges if $p > 0 \to$

Combining the conditions, $\boxed{0 < p \le \frac{1}{2}}$

d) $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ converges if $2p > 1 \to p > \frac{1}{2}$ $\quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ diverges if $p \le 1 \to$

Combining the conditions, $\boxed{\frac{1}{2} < p \le 1}$

admission gate. $\int_{17} E(t)\ dt = 12171.28 \text{ --> } 1271$

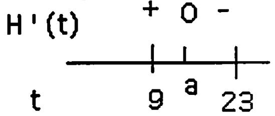
So $(1271)(\$11) + (6004)(\$15) = \boxed{\$104{,}041}$

Way 2 Calculate $\int_{9}^{17} E(t)\ dt \quad - \int_{17}^{23} E(t)\ dt$ and then round off giving a value of $\boxed{\$104{,}048}$

c) $H(t) = \int_{9}^{t} (E(t) - L(t))\ dt \quad 9 < t < 23$

$H'(t) = E(t) - L(t) \quad H'(17) = E(17) - L(17) = \boxed{-380.281}$
$H(17) = 3725$ means that there are 3725 people in the park at 5:00 PM
$H'(17) = -380.281$ means that at 5:00 PM the net change per hour of people in the park was -380.281, or that -380.281 more people left than entered, or the number of the people in the park is decreasing at a rate of 320.281 per hour..

d) $H(t)$ is an absolute maximum on [9,23] at either the endpoints of the interval or where $H'(t) = 0$. $H'(t) = 0 \quad 9 < t < 23$ at $t = 15.795$
Calling that point a, he chart shows it is a relative maximum



Thus the most people are in the park at 15.795 hours or about $\boxed{3{:}48 \text{ PM}}$

AB/BC4

a) $g(-1) = \int_{0}^{-1} f(t)dt$ which is $-\int_{-1}^{0} f(t)dt$ which is the area under the curve $f(t)$ from -1 to 0 times -1 which is $\boxed{\frac{-3}{2}}$

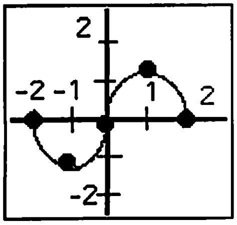
$g'(-1) = f(-1) = \boxed{0}$

$g''(-1) =$ slope of tangent to $f(t)$ at $t = -1 = \frac{3 - -3}{0 - -2} = \boxed{3}$

b) g is increasing when g ' or f is positive. f is positive on $\boxed{-1 < x < 1}$

c) g is concave down when g '' or f ' is negative. f ' < 0 when f is decreasing. f is decreasing on $\boxed{0 < x < 2}$

d)



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BC3

a) $\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$

$y'(2) = (20 - 2))\sin(2) + \cos(2) - 1 = 14.951$

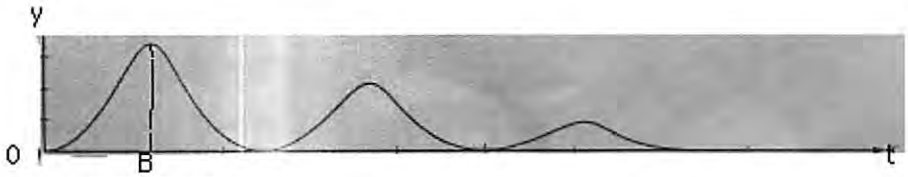
$x'(2) = 10 + 4\cos(2) = 8.335$

slope at $t = 2 = \frac{y'(2)}{x'(2)} = \boxed{1.794}$

b) $x = 140 \dashrightarrow 10t + 4\sin t - 140 = 0 \dashrightarrow t = 13.647 = A$

$\vec{a} = \left(x''(A), y''(A)\right) = \boxed{(-3.529, 1.226)}$

c) Maximum height --> $y(t)$ is a max on $[0,18]$
From the graph of $y(x)$, this occurs at a relative maximum
So--> the graph of $y(t)$ -->



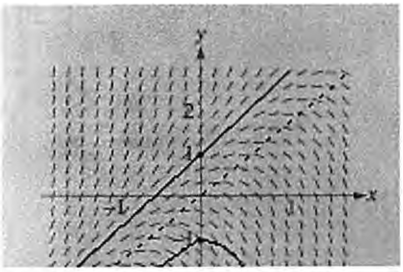
$B = \boxed{t = 3.024}$

Speed $_{t=B} = \left|\vec{v}\right| = \sqrt{(x'(B))^2 + (y'(B))^2} \sim \boxed{6.028\ \frac{m}{sec}}$

d) $y(t) = 0 \dashrightarrow \boxed{t = 6.283 = C \text{ and } t = 12.566 = D}$

Average speed $= \boxed{\frac{1}{D - C}\int_C^D \sqrt{(x'(t))^2 + (y'(t))^2}\, dt}$

BC5



a)

This is just a sketch approximation of the graph.

b) $f(0) = 1 \quad \Delta x = .1$

$f(.1) \sim f(0) + f'(0)(.1) = 1 + (2 - 0)(.1) = 1 + .2 = 1.2$

$f(.2) = f(.1) + f'(.1)(.1) = 1.2 + (2.4 - .4)(.1) = 1.2 + (2)(.1) = \boxed{1.4}$

c) $y = 2x + b \quad \frac{dy}{dx} = 2 = 2y - x$ --> $2y = 4x + 2$ -> $y = 2x + 1$, so $\boxed{b = 1}$

d) $g(0) = 0$

According to the slope field, at $g(0) = 0$, $\frac{dy}{dx} = 0$. Thus, $g(x)$ has a critical point at (0,0). Also, from (0,0), when $x > 0$, $\frac{dy}{dx} < 0$ and where $x < 0$, $\frac{dy}{dx} > 0$.

Thus, $\boxed{(0,0) \text{ is a relative maximum}}$ Alternatively--> According to the slope field, $g'(0) = 0$, so (0,0) is a critical point of g. \F($d^2(y), dx^2) = 2\left(\frac{dy}{dx}\right) - 4$. Since $\frac{dy}{dx} = 0$ at (0,0), then $\frac{d^2(y)}{dx^2} = -4 < 0$.

Therefore g is concave down there. Therefore (0,0) is a relative maximum

BC6

a) $\lim_{n \to \infty} \left| \frac{(2x)^{n+2}}{n+2} \cdot \frac{n+1}{(2x)^{n+1}} \right| = \lim_{n \to \infty} \left(\frac{n+1}{n+2}\right) |2x| = |2x| < 1$ -->

$-1 < 2x < 1$

When $x = \frac{1}{2}$, $\sum \frac{1}{n+1} \quad \int_2^{\infty} \left(\frac{1}{n+1}\right) = {}_2^{\infty}[\ln(n+1)$ --> ∞ --> series diverges

When $x = \frac{1}{2}$, $\sum \frac{(-1)^{n+1}}{n+1}$ alternating series, terms decreasing -->

--> series converging --> $\lim_{n \to \infty} \frac{1}{n+1} = 0$

Interval of convergence: $\boxed{-\frac{1}{2} \le x < \frac{1}{2}}$

b) $f'(x) = 2 + 4x + 8x^2 + 16x^3 + \ldots + 2(2x)^n + \ldots$

$\boxed{\sum_{n=0}^{\infty} 2(2x)^n}$

c) $f'\left(-\frac{1}{3}\right) = 2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \ldots$

This is a geometric series with $a = 2$ and $r = -\frac{2}{3}$ so

$\boxed{S = \frac{2}{1 - \left(-\frac{2}{3}\right)}}$ So $f'\left(-\frac{1}{3}\right) = \boxed{\frac{6}{5}}$

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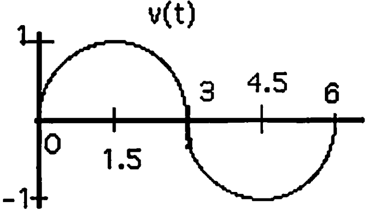
AB 3

a) Given $x(0) = 2$ $v(t) = \sin\left(\frac{\pi t}{3}\right)$ Thus $a(t) = v'(t) = \frac{\pi}{3}\cos\left(\frac{\pi t}{3}\right)$ so

$\frac{\pi}{3}\cos\left(\frac{4\pi}{3}\right) = \frac{\pi}{3}\left(-\frac{1}{2}\right) = \boxed{-\frac{\pi}{6}}$ or $\boxed{-0.524}$

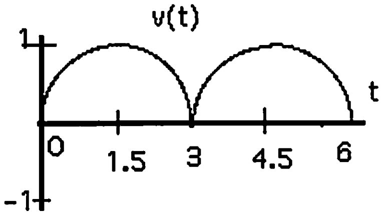
b)



Period $= \frac{2\pi}{\frac{\pi}{3}} = 6$

I. Looking at the graph of $v(t)$, it is decreasing on $3 < t < 4.5$ so consequently the Statement I is $\boxed{\text{true}}$.

II, The speed of the object is the absolute value of the velocity. That graph would look like



and on the interval here, it is increasing, thus making Statement II $\boxed{\text{true}}$.

c) Total distance traveled $= \int_0^4 |v(t)|\,dt$ or $\int_0^3 v(t)\,dt - \int_3^4 v(t)\,dt =$ $\boxed{2.387}$

d) To find the position, you would find the displacement from 0 to 4 of the point and then you would add this to its original position at $t = 0$.

Displacement would be $\int_0^4 v(t)\,dt = x(4) - x(0) = 1.432$ +original position 2

gives position at $t = 4$ of $\boxed{3.432}$

AB5

Note: $\frac{r}{h} = \frac{5}{10}$ $r = \frac{1}{2}h$ $h = 2r$ $V = \frac{1}{3}\pi r^2 h$ $V = \frac{2}{3}\pi r^3$ $V = \frac{1}{12}\pi h^3$

a) $h = 5$ $r = \frac{5}{2}$ $V = \frac{1}{3}\pi\left(\frac{5}{2}\right)^2(5) = \boxed{\frac{125\pi}{12}\text{ cm}^3}$

b) Use $V = \frac{1}{12}\pi h^3$ $\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt} = \frac{\pi}{4}(5)^2\left(\frac{-3}{10}\right) = \boxed{\frac{-15\pi}{8}\ \frac{\text{cm}^3}{\text{hr}}}$

c) Use $V = \frac{2}{3}\pi r^3$ $\frac{dV}{dt} = 2\pi r^2\frac{dr}{dt}$ but, --> $\frac{dh}{dt} = 2\frac{dr}{dt}$ --> $\frac{dr}{dt} = -\frac{3}{20}$

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AB/BC 2

a) $P'(9) = -.646 \to P'(9) < 0$ So function is $\boxed{\text{not increasing}}$

b) $-\dfrac{d\,3e^{(-0.2\sqrt{x})}}{dx} = -3e^{(-0.2\sqrt{x})}\left(\dfrac{-0.2\sqrt{x}}{2\sqrt{x}}\right)$ which is positive for all $x > 0$

Thus $-3e^{(-0.2\sqrt{x})}$ is an increasing function for $x > 0$. Thus the absolute minimum value will happen at 0 or a critical point. $P'(0) < 0$, so the critical point where $p'(t) = 0$ is the absolute minimum value.

$p'(t) = 0$ for $t = \boxed{30.174}$

c) The integral of $P'(t)$ from 0 to t represents the amount of pollutant that has been added or subtracted from the lake over the period of time from 0 to t. You then add the pollutant that was present at t= 0 to get P(n).

$50 + \int_0^{30.174} P'(t)\,dt = P(30.174) = 35.104 \to \boxed{\text{Yes}}$

d) $P - 50 = -2(t - 0) \to P = -2t + 50$ For $P \le 40$ $\boxed{t \ge 5}$

AB/BC 4

a) $g(6) = 5 + \int_6^6 f(t)\,dt = 5 + 0 = \boxed{5}$ $g'(6) = f(6) = \boxed{3}$ $g''(6) = f'(6) = \boxed{0}$

b) f is the derivative of g, so where $f < 0$, g is decreasing.
$\boxed{(-3,0)(12,15)}$

c) $g'' < 0$ means g is concave down. $g'' = f'$ $f' < 0$ when f is decreasing
$\boxed{(6,15]}$

d) $\frac{3}{2}[-1 + 2(0) + 2(1) + 2(3) + 2(1) + 2(0) + (-1)] = \boxed{12}$

AB/BC 5

Note: $\int y\,dy = \int (3 - x)\,dx \to \frac{y^2}{2} = 3x - \frac{x^2}{2} + C$

a) If $y = -2$ is tangent to the graph, then the graph has 0 slope at this point.

$\frac{dy}{dx} = 0 = \frac{3-x}{y}$ Thus $0 = \frac{3-x}{-2} \to \boxed{x = 3}$

In a neighborhood of $y = -2$, $y \le 0$. For $x < 3$, $(3 - x) > 0$, so $\frac{3-x}{y} < 0$
For $x > 3$, $(3 - x) < 0$, so $\frac{3-x}{y} > 0$. This makes the point, (3,−2) a
$\boxed{\text{relative minimum}}$

b) $\int y\,dy = \int (3 - x)\,dx \to \frac{y^2}{2} = 3x - \frac{x^2}{2} + C$ Using $g(6) = -4$

$\frac{16}{2} = 3(6) - \frac{36}{2} + C \to C = 8 \to \frac{y^2}{2} = 3x - \frac{x^2}{2} + 8 \to$

$y^2 = 6x - x^2 + 16 \to$ $\boxed{y = -\sqrt{6x - x^2 + 16}}$ (Note: Choose negative square root so that $g(6) = -4$.)

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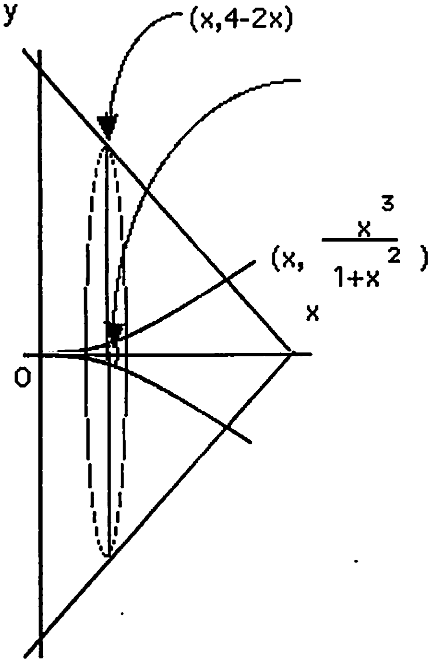
AB1

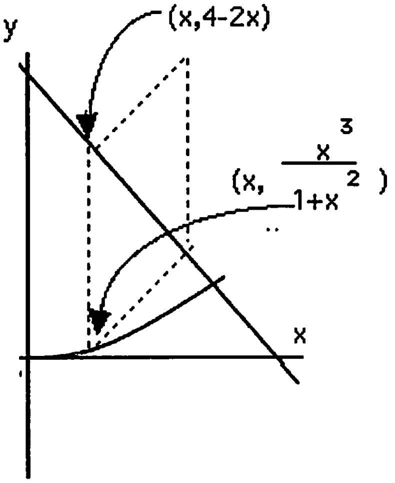
Note: Using graphing calculator to find intersection point of 2 curves, a.

$$4 - 2x - \left(\frac{x^3}{1+x^2}\right) = 0 \quad a = 1.488$$

a) From graph $4 - 2x$ is the upper curve.

$$\int_0^a \left((4-2x) - \left(\frac{x^3}{1+x^2}\right)\right) dx = \boxed{3.215}$$



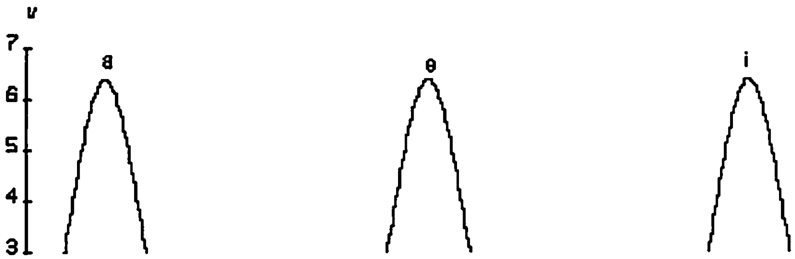


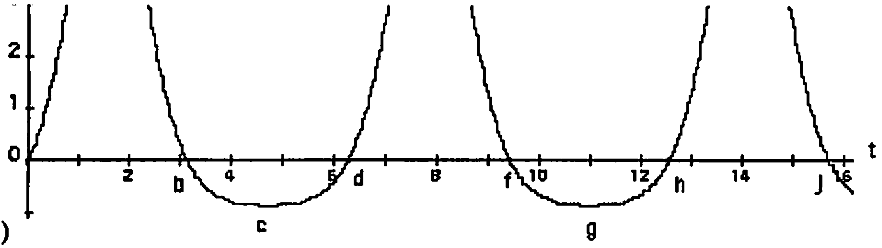
b)

$$\pi \int_0^a (4-2x)^2 - \left(\left(\frac{x^3}{1+x^2}\right)^2\right) dx = \boxed{31.885}$$

c) $$\int_0^a \left(\left((4-2x) - \left(\frac{x^3}{1+x^2}\right)\right)^2\right) dx = \boxed{8.997}$$

AB3





a)

Note: This is periodic, period $= 2\pi$

y-value of max = 6.389 y-value of min = −.865 v(16) = −.438

Table of x-values

zero	max	min
0	a = 1.571	c = 4.713
b = π	e = 7.854	g = 10.996
d= 2π	i = 14.137	
f = 3π		
h = 4π		
j= 5π		

b) This would be when $v(t) < 0$ --> intervals $(\pi,2\pi)(3\pi,4\pi)(5\pi,16)$

c) $\int_0^{\pi} \left(e^{2\sin t} - 1\right)\, dt - \int_{\pi}^{4} \left(e^{2\sin t} - 1\right)\, dt = \boxed{10.542}$

d) The area above the curve from t = 0 on is always greater than the area below the curve.

AB6

a) $\sqrt{3^2+4^2} = \boxed{5\text{ km}}$

b) $x^2+y^2 = z^2$ --> 2x \F(dx,dt) + 2y \F(dy,dt) = 2z \F(dz,dt) --> $2(4)(-15) + 2(3)(10) = 2(5)\frac{dz}{dt}$ -->

$-120 + 60 = 10\,\frac{dz}{dt}$ --> $\frac{dz}{dt} = \boxed{-6\,\frac{\text{km}}{\text{hr}}}$

c) $\tan\Theta = \frac{y}{x}$ Take the derivative with respect to t -->

$\sec^2\Theta\,\frac{d\Theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dy}{dt}}{x^2}$ --> $\left(\frac{5}{4}\right)^2\frac{d\Theta}{dt} = \frac{4(10)-3(-15)}{16}$ --> $\frac{d\Theta}{dt} = \left(\frac{85}{16}\right)\bullet\left(\frac{16}{25}\right) =$

$\boxed{\frac{d\Theta}{dt} = \frac{17}{5}\,\frac{\text{radians}}{\text{hr}}}$

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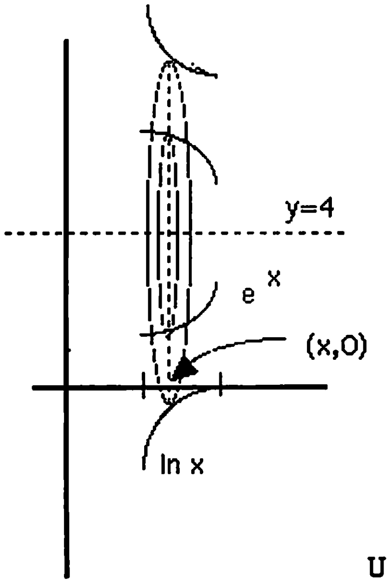
AB/BC1

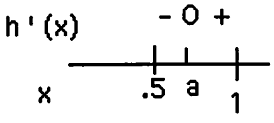
a) Looking at the graph, $e^x > \ln x$ for $\frac{1}{2} < x < 1$ $\int_{1/2}^{1} (e^x - \ln x)\ dx = \boxed{1.223}$



b) Using discs $\pi \int_{1/2}^{1} (4 - \ln x)^2 - (4 - e^x)^2\, dx =$ $\boxed{23.609}$

c) Absolute maximum and absolute minimum of a continuous function on a closed interval happen at either and endpoint or a relative maximum or relative minimum point.

$h(x) = e^x - \ln x$ $h'(x) = e^x - \frac{1}{x}$
$h(.5) = 2.343$ $h(1) = 2.718$



For $.5 < x < 1$ $h'(x) = 0$ at $a = 0.567$ so h ' (a) is a relative minimum $h(a) = 2.330$
Therefore $\boxed{\text{abs max} = h(1) = 2.718 \ \ \text{abs min} = h(a) = 2.330}$

AB/BC2

a) Number of people = $\int_{9}^{17} E(t)\ dt = 6004.27 \text{ --> } \boxed{6004}$

b) Two ways to think about this.
Way 1 Calculate number of people admitted to the park for period from 9:00 AM until 5:00 PM --> $\int_{9}^{17} E(t)\ dt$ and then calculate the number of people admitted to the park for period from 5:00 PM to 11:00 PM ---> $\int_{17}^{23} E(t)\ dt$. Round each off to nearest whole number. Then calculate

23

Thus $2\pi r^2 \frac{dr}{dt} = 2\pi r^2 \frac{-3}{20} = \frac{-3}{10}\pi r^2$

The exposed surface area at any time is $S = \pi r^2$

Setting them as a direct proportion $\frac{-3}{10}\pi r^2 = k\pi r^2$ --> $\boxed{k = -\frac{3}{10}}$

AB6

Note: Given table and $f''(x) > 0$

a) Integrating gives ${}_{0}^{1.5}\left[3f(x) + 4x\right. = 3f(1.5) + 4(1.5) - [(3f(0) + 4(0)] =$

$3(-1) + 6 - 3(-7) + 0 = \boxed{24}$

b) $f(1) = -4$, $f'(1) = 5$ --> $\boxed{y + 4 = 5(x - 1)}$

Using $dy = f'(x)dx$ $dy = 5(.2)$ --> $y(1.2) \sim 1 + (-4) = \boxed{-3}$

Given that f'' is always positive means that the curve is concave up, so the tangent line approximation is less than the actual value of $f(1.2)$

c) By the Mean Value Theorem, $f''(c) = \frac{f'(.5) - f'(0)}{.5 - 0} = \frac{3 - 0}{.5} = \boxed{6 = r}$

d) It is not possible. $g'(x) = \begin{cases} 4x - 1 & x < 0 \\ 4x + 1 & x \geq 0 \end{cases}$

Thus $\lim\limits_{x \to 0^+} g(x) \neq \lim\limits_{x \to 0^-}$ --> therefore $g'(0)$ does not exist.

Since $f'(0)$ exists they are not the same function

The College Board AP® ADVANCED PLACEMENT PROGRAM®

2001 REPORT TO AP® TEACHERS

CALCU[illegible] AB

VACAVILLE HIGH SCHOOL

The following charts indicate the number of AP candidates from your school/district in each fifth of the total candidate group for each category described.

AP GRADES	GRADE DISTRIBUTIONS	
	GLOBAL	GROUP
5	16.0%	46.2%
4	23.3%	30.8%
3	24.6%	19.2%
2	17.3%	3.8%
1	18.8%	0.0%
TOTAL CANDIDATES	145434	26

MULTIPLE-CHOICE

	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN
DIFFERENTIAL CALCULUS (29 Q.)		2	4	8	12	29	12.7	19.2
INTEGRAL CALCULUS (16 Q.)	1	1	4	8	12	16	5.9	9.4
PART A- NO CALCULATOR (28 Q.)		2	2	8	14	28	13.0	19.8
PART B- WITH GRAPHING CALCULATOR (17 Q.)	1	3	3	9	10	17	5.7	8.7
WEIGHTED M-C SCORE PERCENT IN EACH FIFTH		2 8%	5 19%	6 23%	13 50%	54	22.3	34.3
	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN

FREE-RESPONSE

	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN
AREA AND VOLUME	3	3	4	5	11	9	4.8	6.3
MODEL NUMERICALLY/ANALYTICALLY- AB & BC	3		5	5	13	9	3.5	5.5
VELOCITY -ACCELERATION -AB & BC	1	1	5	14	5	9	2.4	3.5
TANGENT LINES/ANAL.OF H GIVEN H'-AB & BC	1	1	6	4	14	9	3.1	5.2
ANALYSIS OF FUNCTION/DEFINITE INTEGRAL			11	8	7	9	6.1	8.4
SEPARABLE DIFFERENTIAL EQUATION	1	1	2	7	15	9	3.6	6.8
WEIGHTED F-R SCORE PERCENT IN EACH FIFTH	1 4%	1 4%	3 12%	7 27%	14 54%	54	23.5	35.7
	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN

AI053595
VACAVILLE HIGH SCHOOL

100 MONTE VISTA AVENUE
VACAVILLE CA 95688

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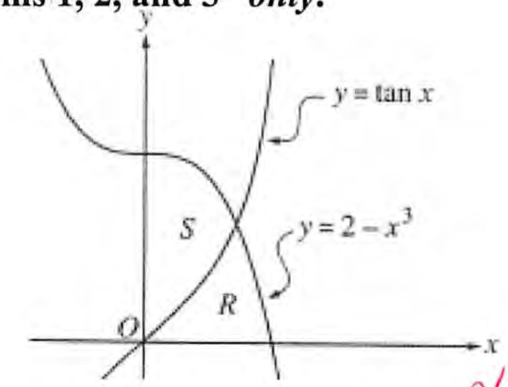
A graphing calculator is allowed on problems 1, 2, and 3 *only.*

1. Let R and S be the regions in the first quadrant as shown in the figure. The region R is bounded by the x-axis and the graphs of $y=2-x^3$ and $y=\tan x$.

   a) Find the area of R.

   b) Find the area of S.

   c) Find the volume of the solid generated when S is revolved about the x–axis.



2. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable W of time t. The table on the right shows the water temperature as recorded every 3 days over a 15-day period.

a plus sign

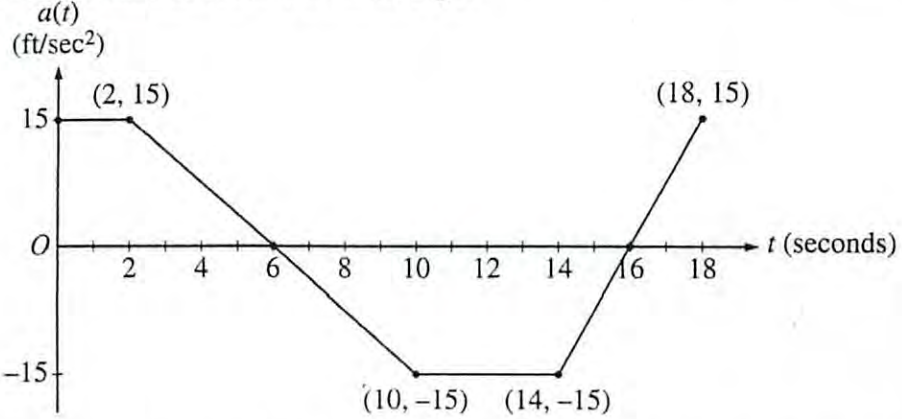
   a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

   b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

   c) A student proposes the function P, given by $P(t)=20+10te^{(-\frac{t}{3})}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

   d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21



3. A car is traveling on a straight road with velocity 55 ft/sec at time $t=0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in $ft/\sec^2$, is the piecewise linear function defined by the graph above.

   a) Is the velocity of the car increasing at $T=2$ seconds? Why or why not?

   b) At what time in the interval $0 \le t \le 18$, other than $t=0$, is the velocity of the car 55 ft/sec? Why?

   c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

   d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

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No calculators allowed on the remaining parts of this examination.

4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

   a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

   b) On what intervals, if any, is the graph of h concave up? Justify your answer.

   c) Write an equation for the line tangent to the graph of h at $x = 4$.

   d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

5. A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

   where a, b, and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point on inflection at $x = -2$.

   a) Find the values of a and b.

   b) If $\int_0^1 f(x)\, dx = 32$, what is the value of k?

6. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\dfrac{dy}{dx} = y^2(6 - 2x)$.

   a) Find $\dfrac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

   b) Find $y = f(x)$ by solving the differential equation $\dfrac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \dfrac{1}{4}$.

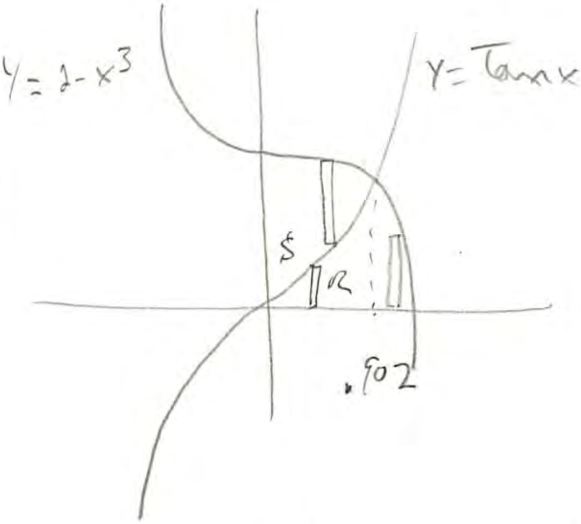
2001

2001

I	c	8.332
II	D	~~27.757~~ 14.243
III	c	Abs max (6, 115)
IV	c	$y = \frac{7}{2}x - 17$
V	b	$k = 5$
VI	b	$y = \frac{1}{x^2 - 6x + 13}$

Both right	+
one right	✓
Tried Both	0 or $\frac{1 \text{ right}}{1 \text{ mistaken}}$
Tried only one	—
Nothing	=

①



$2-x^3 \overset{\text{set}}{=} \tan x$ [illegible]

$x = .902$ ①

$y = 2-x^3 \overset{\text{set}}{=} 0$

① $x = \sqrt[3]{2}$

Draw cross sections ① [illegible]

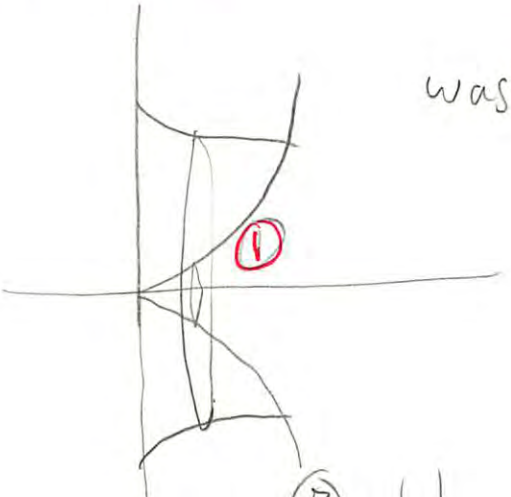
Limits

⑤ A) $\int_0^{.902} \tan x \, dx + \int_{.902}^{\sqrt[3]{2} = 1.26} 2-x^3 \, dx$ Proper integral ① (with limits found)

$.478 + .251 = .729$ ①

④ B) $\int_0^{\boxed{.902}} (\underbrace{2-x^3 - \tan x}_{①}) dx = 1.161$ Limits ① ① cross section ①

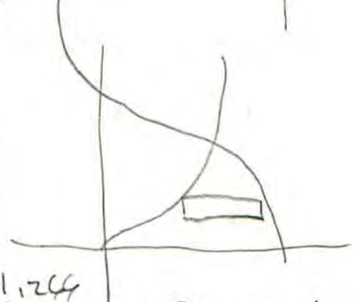
6 c) washers. ① limits ① integrand ②



$\pi \int_0^{.902} (\underbrace{(2-x^3)^2 - \tan^2 x}) dx$

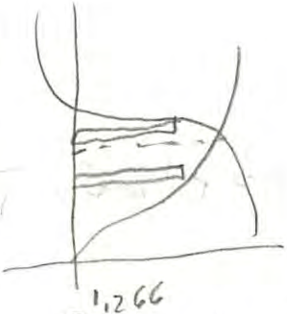
$= 8.332$ ①

Ⓐ



$\int_0^{1.266} \sqrt[3]{2-y} - \tan^{-1} y \, dy = .729$

Ⓑ

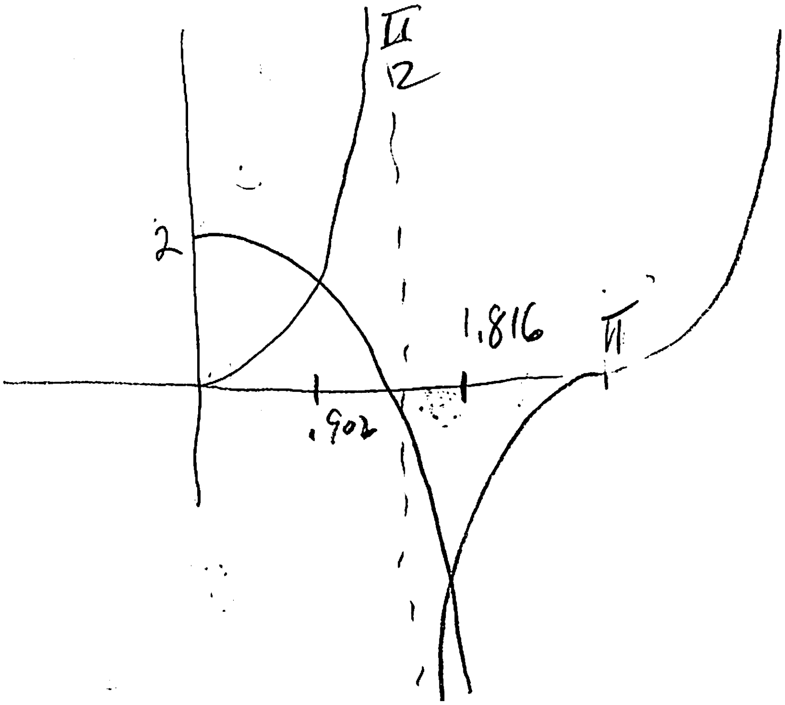


$\int_0^{1.266} \tan^{-1} y \, dy + \int_{1.266}^{2} \sqrt[3]{2-y} \, dy = 1.161$

b

over for side ~~note~~.

2001 ①



If student is not careful about solving

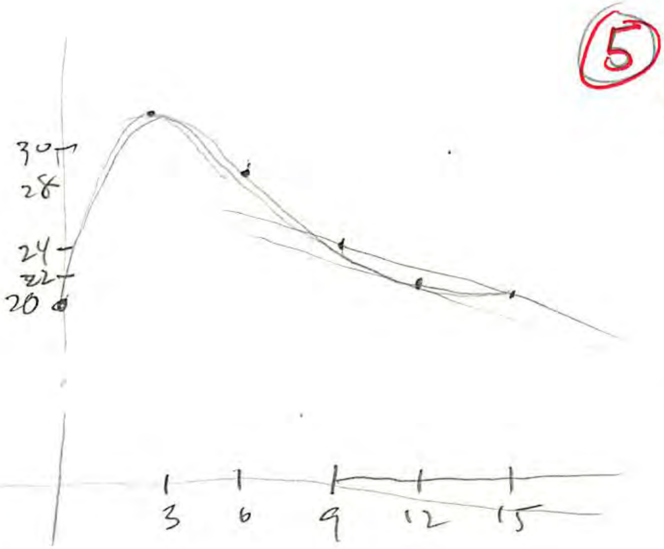
Solve(Tan x − (2 − x ^ 3), x, ___)

Put in a 2 and get 1.816
Put in a 1 and get .902

This focuses the student on
the implied domain $x \in [0, \underset{1.260}{\underset{\uparrow}{\sqrt[3]{2}}}]$

②

t	$W(t)$
0	20
3	31
6	28
9	24
12	22
15	21



(4) ④

(c) $P(t) = 20 + 10\overset{u}{t}\overset{v}{e}^{\left(-\frac{t}{3}\right)}$

$P'(t) = 10e^{-\frac{t}{3}} - \frac{10t}{3}e^{-\frac{t}{3}}$

$P'(12) = 10e^{-4} - \frac{10}{\cancel{3}}\overset{4}{\cancel{12}}e^{-4}$ ①

$\frac{10}{e^4}(1-4)$

$= -\frac{30}{e^4}$ ①

The water is cooling at $\frac{30}{e^4}$ °C per day on the 12th day. ②

(.549)

③ (a) $W'(12)$

① we want the slope of the curve evaluated at $t=12$

$\frac{21-24}{15-9} = \frac{-3}{6} = -\frac{1}{2}$ ②

The average temperature decreases ½ °C per day

⑤ (b) ③

$\frac{3}{2}(20 + 2\cdot31 + 2\cdot28 + 2\cdot24 + 2\cdot22 + 21)$

$\frac{3}{2}(20 + 62 + 56 + 48 + 44 + 21$

$\frac{3}{2}\cdot\frac{251}{1} = 376.5$

Average value $\frac{1}{b-a}\int_a^b f(x)\,dx$ ①

$\frac{1}{15}\cdot 376.5 = 25.1$ ①

③

D) $\frac{1}{15}\int_0^{15} P(t)\,dt$ ②

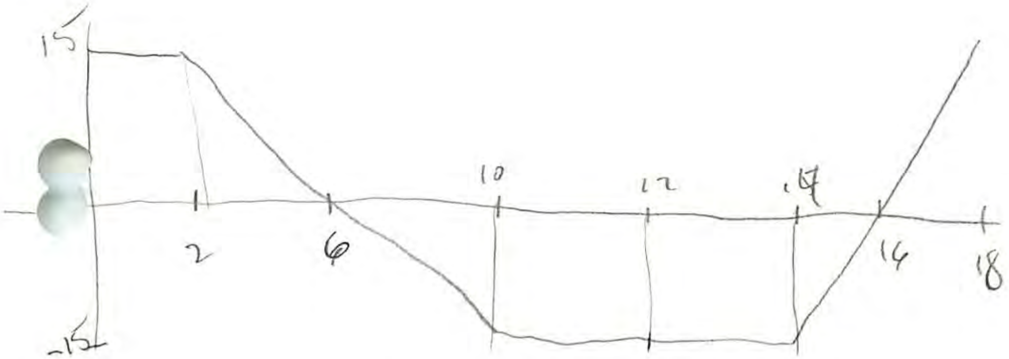
①

$= 25.757$

②

2001

②



$t=0 \quad v=55, \quad a=15$

③

③ A) $V' = a$ since $a > 0$, $V' > 0$ ∴ V is increasing at $t = 2$ Reason ② ①
Since $V(0) > 0$ and acceleration is positive, V is even more POSITIVE.

③ B) V is 55 again when $t = 12$ because

$55 + \text{INT}(a(t), t, 0, 6) + \text{int}(a(t), t, 6, 12)$
$55 + 60 - 60 = 55$

⑤ C) $V' = f' = a$

$+ \;\; 0 \;\; - \;\; ① \;\; 0 \;\; +$
6 16

$t = 6$ is a critical value of $V(t)$ and a max f'test. ②

$t = 18$ max EndPT inc to it.

5 int$(a(t), t, 0, 6)$ = 55 + 60 = 115 ft/sec. ①

(6, 115) ① and (18, 25)

∴ Abs max is at (6, 115)

④ d)

30 30 15
2 6 30 10 30 12 30 14 15 16 18

55+30 55+60 55+30 55 55−30 55−45 55−30
$V = 85 \quad 115 \quad 85 \quad 55 \quad 25 \quad \boxed{10} \quad 25$

The slowest velocity is 10 ft/sec at $t = 16$.
Thus, the velocity will never be zero.

2001
③

④

$h(4) = -3 \qquad h'(x) = \frac{x^2-2}{x} \quad \forall x \neq 0$

①

$$\begin{array}{c|ccccccc} h' & - & 0 & + & u & - & 0 & + \\ \hline & & -\sqrt{2} & & 0 & & \sqrt{2} & \end{array}$$

④ a) ④ $\frac{x^2-2}{x} \overset{\text{set}}{=} 0$

$\frac{(x-\sqrt{2})(x+\sqrt{2})}{x} = 0$

There are horizontal tangents at ③ $x = -\sqrt{2}$ and at $x = \sqrt{2}$.
Both are local min by 1st test.

⑤ ⑤ b) $h' = \frac{x^2-2}{x} \begin{matrix} u \\ v \end{matrix}$

③ h is concave up $\forall x \neq 0$ since $h'' > 0$.

$h'' = \frac{x(2x) - (x^2-2)}{x^2}$

$2x^2 - x^2 + 2$

② $\frac{x^2+2}{x^2} > 0 \quad \forall x \neq 0$

D) The tangent line is below the curve since h is concave up at $x = 4$,

③

③ ③ c) $(4, -3)$ gives

$h'(4) = \frac{16-2}{4}$

$\frac{7}{2} = m$ ①

$y + 3 = \frac{7}{2}(x-4)$

$y = \frac{7}{2}x - 17$ ②

2001

④

⑤

⑩ ⑩

a) $f(x) = 4x^3 + ax^2 + bx + K$ min at $x = -1$

$f'(x) = 12x^2 + 2ax + b$ PI $x = -2$

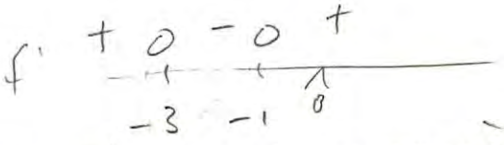
$f''(x) = 24x + 2a$ $f'(-1) = 12 + 2a + b$

$2(12x + a) \overset{\text{set}}{=} 0$ $12 - 48 + b$ ③

③ $-24 + a = 0$ $\therefore \underline{\underline{b = 36}}$

$\underline{\underline{a = 24}}$

$f(x) = 4x^3 + 24x^2 + 36x + K$



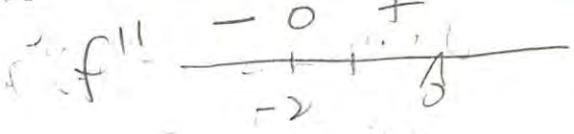
$= 12x^2 + 48x + 36$

$12(x^2 + 4x + 3)$

$12(x + 1)(x + 3)$

min at $x = -1$ as required. ✓

verified that f met conditions ④



$f'' = 24x + 48$

$24(x + 2)$

(P)

⑤ b) $\int_0^1 f(x)\,dx = 32$

$\int_0^1 4x^3 + 24x^2 + 36x + K\,dx \overset{\text{set}}{=} 32$

②

$x^4 + 8x^3 + 18x^2 + Kx \Big|_0^1 = 32$

$1 + 8 + 18 + K = 32$

$K = 5$ ②

2001

⑤

⑥

$(3, \tfrac{1}{4}) \qquad \frac{dy}{dx} = \overset{u}{y^2}\overset{v}{(6-2x)}$

a) ⑦

$\left(\frac{dy}{dx}\right)' = (6-2x)2y\frac{dy}{dx} + y^2(-2)$ ④

$= (6-2x)2y\left(y^2(6-2x)\right) - 2y^2$

$= (6-2x)^2 2y^3 - 2y^2$

$2y^2\left((6-2x)^2 y - 1\right)$ ②

$\left.\frac{d^2y}{dx^2}\right|_{(3,\frac{1}{4})} = 2\cdot\frac{1}{16}\left((6-6)^2\left(\tfrac{1}{4}\right) - 1\right)$

$\frac{1}{8}(-1) = -\frac{1}{8}$ ①

b) ⑧

$\int \frac{dy}{y^2} = \int 6-2x \, dx$

$-\frac{1}{y} = 6x - x^2 + C$ ②

$\frac{1}{x^2-6x+C} = y$

$\frac{1}{9-18+C} \overset{\text{set}}{=} \frac{1}{4}$ ②

$\frac{1}{-9+C} = \frac{1}{4}$ ②

$C = 13.$

so $y = \frac{1}{x^2-6x+13}$ ②

If they end up

$-\frac{1}{y} = 6x - x^2 + C$

$y = \frac{1}{x^2-6x} + C$

and finish the problem $C = \frac{13}{36}$

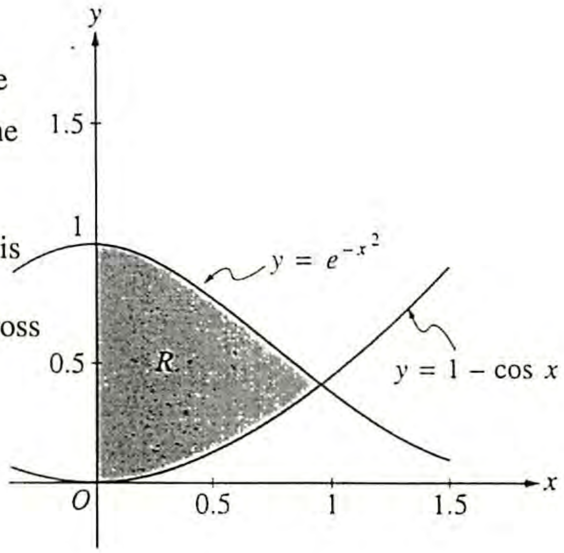
2001

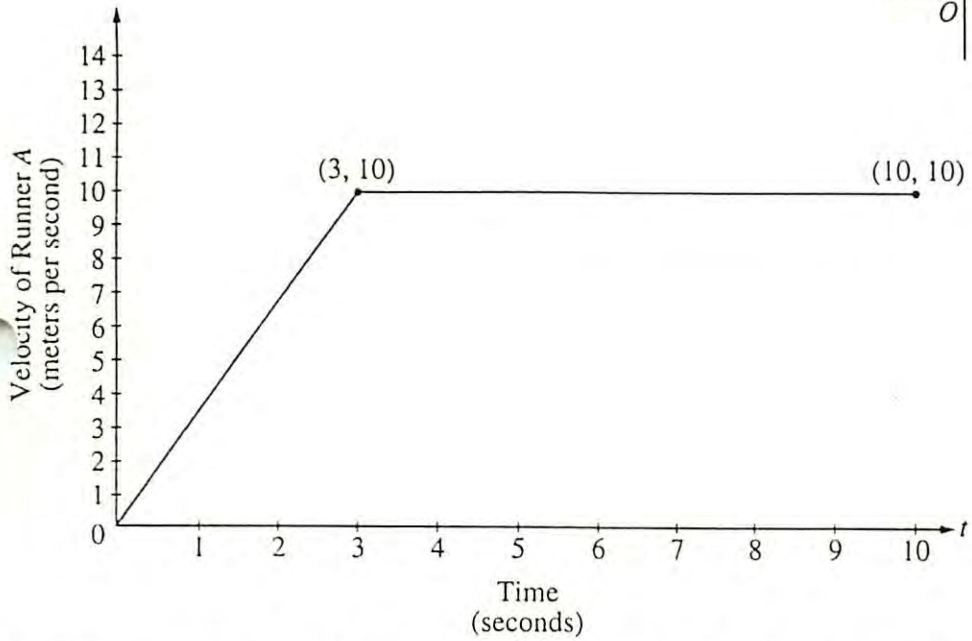
⑥

2000 Advanced Placement *AB* Exam

e^{-x^2}

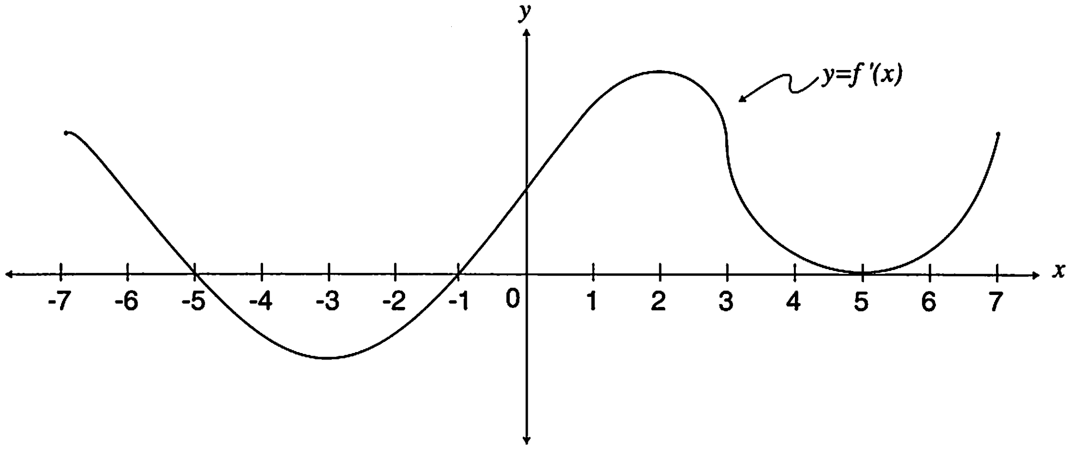
1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure on the right.
   a) Find the area of the region R.
   b) Find the volume of the solid generated when the region R is revolved about the x-axis
   c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.





2. Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.
   a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.
   b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.
   c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

2000



3. The figure above shows the graph of $f'(x)$, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.
   a) Find all values of x, for $-7 < x < 7$, at which f attains a relativeminimum. Justify your answer.
   b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.
   c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.
   d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

4. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.
   a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?
   b) How many gallons of water are in the tank at time $t = 3$ minutes.?
   c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.
   d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

5. Consider the curve given by $xy^2 - x^3y = 6$.
   a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.
   b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.
   c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

6. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{3x^2}{e^{2y}}$.
   a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \dfrac{1}{2}$.
   b) Find the domain and range of the function f found in part (a).

2000

200-C

Both right +

one right ✓

Tried both 0

Tried one —

Nothing =

① b 1,747
or 1.746

② b $\frac{72}{49} = 1.469$ AND $\frac{10}{3} = 3.333$

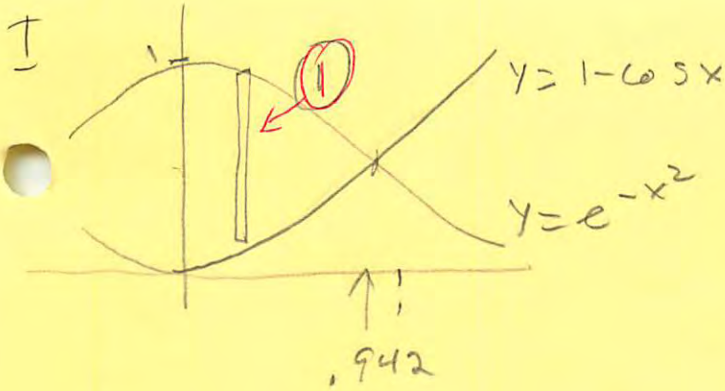
③ c $-7 < x < -3$
AND $2 < x < 5;\ x \neq 3$

④ d max at $t = 63$ f'' test

⑤ c only vertical tangent $x = (-24)^{\frac{1}{5}}$

⑥ a) $f(x) = \frac{1}{2} \ln(2x^3 + e)$

I



2pts → .942 area/volume

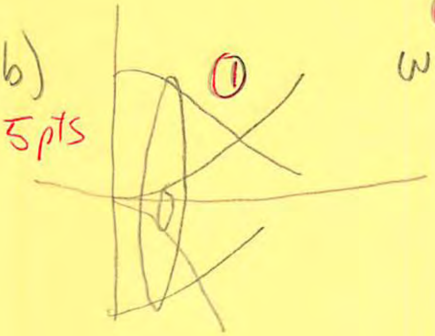
a) 6pts

$$\int_0^{.942} e^{-x^2} - (1-\cos x)\,dx$$ ②

math 0 Solve(Y1, X, 1) = .942 -> A

math 9 fnInt(Y1, X, 0, A) = .591 ①

b) 5pts



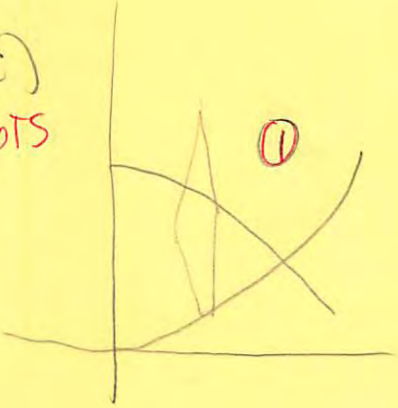
① washers

$$\pi\int_0^{.942} e^{-2x^2} - (1-\cos x)^2\,dx$$ ②

$$(Y_1)^2 - (Y_2)^2$$ ①

fnInt($Y_1^2 - Y_2^2$, X, 0, A) * π = 1.747

c) 4pts



$$\int_0^{.942} \left[e^{-x^2} - (1-\cos x)\right]^2 dx =$$ ②

.461 ①

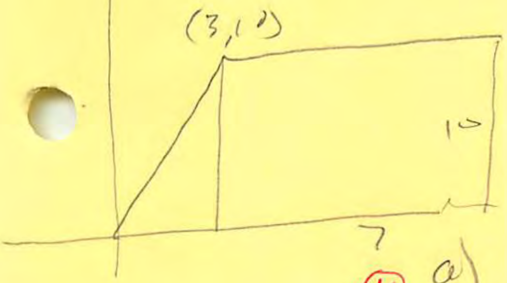
I A $\int_0^1$ etc = .588
Didn't compute proper intersection

B Same error leads to
1.740 answer.
.554 if they forget to
multiply by π

C using .942 or 1 yields .461

II

Velocity fcn A



Velocity acceleration distance

$$v_A = \begin{cases} \frac{10}{3}t & \text{if } 0 < t < 3 \\ 10 & \text{if } 3 \le t \le 10 \end{cases} \qquad v_B = \frac{24t}{2t+3}$$

④ a) $v_A\Big|_{t=2} = \frac{20}{3}$ m/sec ① ① 6.667 $\qquad v_B\Big|_{t=2} = \frac{48}{7}$ ① m/sec ① 6.857

⑤ b) acceleration is der of v ①

$a_A = \frac{10}{3}$ ① m/sec^2 ① 3.333 $\qquad a_B = \left(\frac{24t}{2t+3}\right)'$

$$\frac{(2t+3)24 - 24t \cdot 2}{(2t+3)^2}$$

$48t + 72 - 48t$

$$a_B \quad \frac{72}{(2t+3)^2}\Big|_{t=2}$$

$\frac{72}{49}$ ① m/sec^2 ①

1.469

⑥ c) $D_A = \int_0^3 \frac{10}{3}t\,dt + \int_3^{10} 10\,dt$ ①

$\frac{1}{2} \cdot 3 \cdot 10 + 70$

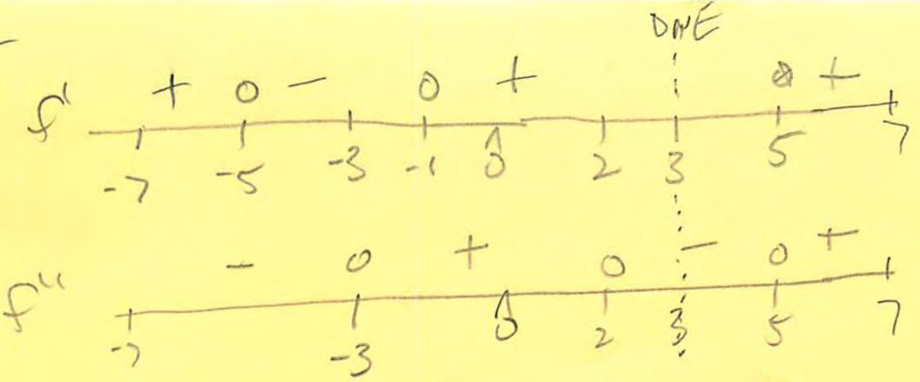
$15 + 70$

85 ① meters ①

D_B ① $= \int_0^{10} \frac{24t}{2t+3}\,dt = 83.336$ ① meters ①

III

Extreme Value pblm



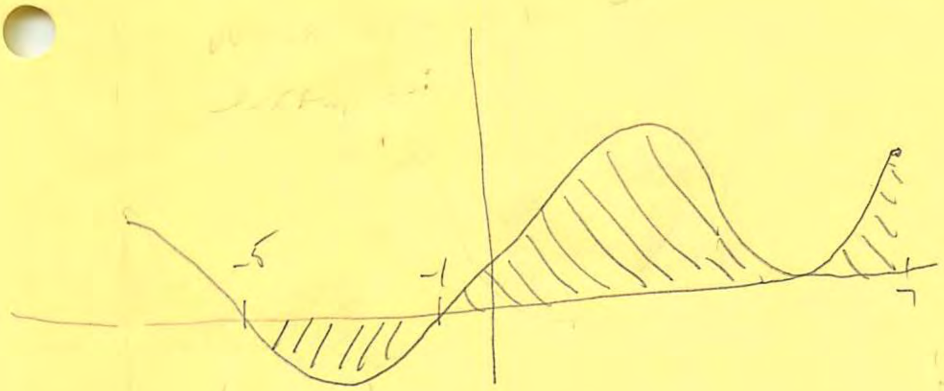
③ a) f has min at x ② = −1 f' test ①

③ b) f has max at x ② = −5 f' test ①

④ c) $f'' < 0$ when f' is decreasing ① ∴ $-7 < x < -3$ ① and $2 < x < 5$ ① ; $x \neq 3$ ①

f'' cannot be less than zero if it DNE.

⑤ d) rel max at: x = −5 f' test
x = 7 End pt inc to it.



$$\int_{-5}^{7} f'(x) = f(7) - f(-5)$$ By FTC

since the area bounded between curve and x axis from x = −5 to x = −1 is less than the area bounded from x = −1 & x = 7 $f(7) - f(-5) > 0$ so $f(7) > f(-5)$ ∴ f(7) is absolute max.

Answer must have Justifications ⑤

IV

$0 \le t \le 120$ $T=0$ $V=30$

Related Rates

$\frac{dv}{dt}=8$

leak $\frac{dl}{dt}=-\sqrt{t+1}$

③ a) $\int_0^3 \sqrt{t+1}\,dt$ ①

$\frac{2(t+1)^{\frac{3}{2}}}{3}\Big|_0^3 = \frac{2}{3}(8-1)$

$= \frac{14}{3}$ ①

$4\frac{2}{3}$ gals leak out in 3 minutes.

Proper sentence ①

③ b) 30 gals

+24 gal ① method

$-\ 4\frac{2}{3}$ gal

$\frac{148}{8}$

$49\frac{1}{3}$ gal in tank at $t=3$ ①

①

④ c) $A(t) = 30 + 8t - \int_0^t \sqrt{x+1}\,dx$ ④

OR

$A(t) = 30 + 8t - \frac{2}{3}(t+1)^{\frac{3}{2}} + \frac{2}{3}$

$8t - \frac{2}{3}(t+1)^{\frac{3}{2}} + \frac{92}{3}$

⑤ d) $A'(t) = 8 - \sqrt{t+1} \overset{\text{set}}{=} 0$ ② ①

$\sqrt{t+1} = 8$

$t+1 = 64$

$t = 63$

$A''(t) = -\frac{1}{2\sqrt{t+1}}$ <0 for $t=63$..!, max at $t=63$. f'' test. ① ①

II $\overset{u\ v}{xy^2} - \overset{u\ v}{x^3y} = 6$ Implicit

③ a) $y^2 + 2xy\frac{dy}{dx} - \left(3x^2y + x^3\frac{dy}{dx}\right) = 0$ ②

$(2xy - x^3)\frac{dy}{dx} = 3x^2y - y^2$

$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$ ①

⑦ b) $x = 1$

$y^2 - y = 6$

$y^2 - y - 6 = 0$

② $(y-3)(y+2) = 0$

$y \in \{3, -2\}$

Points are $(1,3)$ and $(1,-2)$ ①

$\left.\frac{dy}{dx}\right|_{(1,3)} = \frac{3\cdot1\cdot3 - 9}{2\cdot1\cdot3 - 1} = 0$ ①

$\left.\frac{dy}{dx}\right|_{(1,-2)} = \frac{3\cdot1\cdot(-2) - 4}{2(1)(-2) - 1}$

$\frac{-10}{-5} = 2$ ①

$y - 3 = 0(x-1)$

$y = 3$ ①

$y + 2 = 2(x-1)$

$y = 2x - 4$ ①

⑤ c) vertical tangent when $\frac{dy}{dx}$ DNE ∴ ① $2xy - x^3 \overset{set}{=} 0$

$x(2y - x^2) = 0$

$x = 0$ $\quad 2y = x^2$

$y = \frac{x^2}{2}$ ①

$x \cdot \frac{x^4}{4} - x^3 \cdot \frac{x^2}{2} = 6$

① $x = 0$ is rejected, in part "a" $x = 0$ would have allowed division by zero so $x \neq 0$

$\frac{x^5}{4} - \frac{x^5}{2} = 6$

$\frac{x^5 - 2x^5}{4} = 6$

$-x^5 = 24$

① $x = -24^{\frac{1}{5}}$ ∴ only vertical tangent when $x = (-24)^{\frac{1}{5}}$ ①

VI $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$

differential. Separation of variables.

a) ⑩

$\int e^{2y}\,dy = \int 3x^2\,dx$ Separate variables show integral ②

$\frac{1}{2}e^{2y} = x^3 + C \longleftarrow \begin{cases} \text{integrate ①} \\ \text{has } C \text{ ①} \end{cases}$

$e^{2y} = 2x^3 + 2C$

$2y = \ln(2x^3 + 2c)$ Eliminate exp ②

$y = \frac{1}{2}\ln(2x^3 + 2c) = f(x)$

$f(0) = \frac{1}{2}\ln 2c \overset{set}{=} \frac{1}{2}$ ← ①

$\ln 2c = 1$

$\therefore 2c = e$

$c = \frac{e}{2}$ ①

$f(x) = \frac{1}{2}\ln(2x^3 + e)$ ②

b) ⑤

$2x^3 + e > 0$ since Domain of ln is > 0. ①

$x^3 > -\frac{e}{2}$

$x > \left(-\frac{e}{2}\right)^{\frac{1}{3}}$ $\therefore D_f$ $x > \left(-\frac{e}{2}\right)^{\frac{1}{3}}$ ②

Range $f(x) \in \mathbb{R}$ ②

From: Doug Kuhlmann <dkuhlmann@andover.edu>
To: AP-Calculus <ap-calculus@lyris.collegeboard.com>
Cc: Jim <teeitup98@telocity.com>
Date: Friday, April 19, 2002 3:03 AM
Subject: Re: 2000 AB6

Replyu below:

>We were working on the 2000 AB test. For problem 6 :
>
>Problem 6:
>
>Consider the differential equation: (dy/dx) = (3x^2) / e^(2y)
>
>a) Find a solution y = f(x) satisfying f(0) = .5
>b) Find the domain and range of f.
>
>the rubric shows the "a" answer as:
>
>Y = .5 ln (2x^3 + e)
>
>A student asked why the absolute value bars were left off?
That would make
>the answer:
>
> Y = .5 ln | 2x^3 + e |
>
>The reason this makes a difference is actually in the next
part of the
>problem where the domain of the function described.
>
>If the absolute value bars are included we thought the domain
should have
>been
>
>X not equal cube root (- e / 2) instead of the answer :
>
>X > cube root (- e / 2).
>
>What are we missing?
>
>TIA

>
\>Jim Mumaugh
>
\>Vacaville High School

Hi Jim,

When solving the given differential equation and using the initial condition, you first get something equivalent to e^(2y) = 2x^3 + e. Note that since e^(2y) is positive for any y, x is restricted to make 2x^3+e>0. The domain restriction for f(x) comes at this stage. We don't need absolute value because is implied earlier that 2x^3+e must be positive. HTH.

Doug
--
Doug Kuhlmann
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Phillips Academy
180 Main St.
Andover, MA 01810
dkuhlmann@andover.edu

From: LnMcmullin@aol.com
To: <ap-calculus@lyris.collegeboard.com>, <teeitup98@telocity.com>
Date: Friday, April 19, 2002 3:57 AM
Subject: Re: [ap-calculus] 2000 AB6

In a message dated 4/19/2002 6:38:16 AM Eastern Daylight Time, teeitup98@telocity.com writes:

> We were working on the 2000 AB test. For problem 6 :
>
> Problem 6:
>
> Consider the differential equation: (dy/dx) = (3x^2) / e^(2y)
>
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> If the absolute value bars are included we thought the domain should have been
>
> X not equal cube root (- e / 2) instead of the answer :
>
> X > cube root (- e / 2).
>
> What are we missing?

The equation that you get after separating the variables and integrating is e^(2y) = 2x^3 + C (third line on the scoring standard). The left side here is positive for all y. Thus the right side must be positive. The right side becomes the argument of the logarithm and since it is positive there is no need to use absolute value, nor is there a need to consider in part (b) 2x^3 + e < 0.

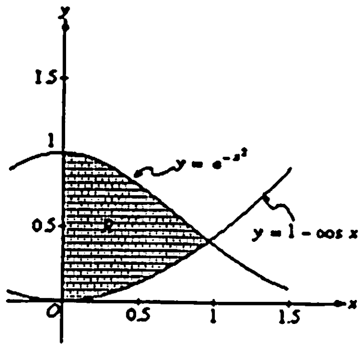
When integrating f ' (x) / f(x) * dx it is necessary to use absolute value ln | f(x) | + C. This does not mean that every time a logarithm shows up it needs absolute value signs. The logarithms in this problem did not appear as the result of integrating f ' (x) / f(x) * dx. Putting them in, as your question points out, changes the domain. Consider what would happen if the problem resulted in - e^(2y) = 2x^3 + C (note new negative sign on left side). Then the domain would be found by analyzing 2x^3 + C < 0, not by making everything positive with the absolute value sign.

Hope this helps.

Lin

Lin McMullin
Niantic, CT.

AB–1 / BC–1 FINAL DRAFT FOR SCORING 2000



Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

Region R

$e^{-x^2} = 1 - \cos x$ at $x = 0.941944 = A$

1 : Correct limits in an integral in (a), (b), or (c).

(a) area $= \int_0^A (e^{-x^2} - (1 - \cos x))\,dx$

$= 0.590$ or 0.591

$$2\begin{cases}1: \text{ integrand}\\ 1: \text{ answer}\end{cases}$$

(b) Volume $= \pi \int_0^A \left(\left(e^{-x^2}\right)^2 - (1 - \cos x)^2\right)dx$

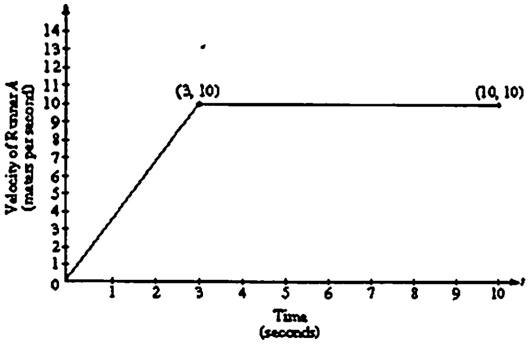
$= 0.55596\pi = 1.746$ or 1.747

$$3\begin{cases}2: \text{ integrand and constant}\\ \quad <-1> \text{ each error}\\ 1: \text{ answer}\end{cases}$$

(c) Volume $= \int_0^A \left(e^{-x^2} - (1 - \cos x)\right)^2 dx$

$= 0.461$

$$3\begin{cases}2: \text{ integrand}\\ \quad <-1> \text{ each error}\\ \quad \text{note: 0/2 if not of the form}\\ \qquad k\int_c^d (f(x) - g(x))^2\,dx\\ 1: \text{ answer}\end{cases}$$



Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.

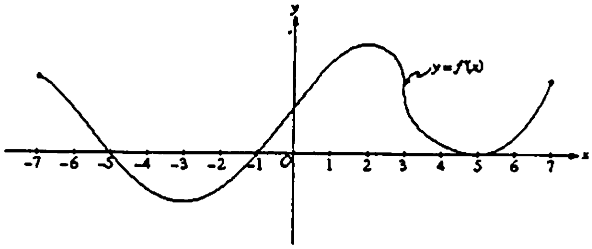
(a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

(b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

(a) Runner A: velocity $= \dfrac{10}{3} \cdot 2 = \dfrac{20}{3}$ $= 6.666$ or 6.667 meters/sec Runner B: $v(2) = \dfrac{48}{7} = 6.857$ meters/sec	$2\begin{cases} 1: \text{ velocity for Runner } A \\ 1: \text{ velocity for Runner } B \end{cases}$
(b) Runner A: acceleration $= \dfrac{10}{3} = 3.333$ meters/sec^2 Runner B: $a(2) = v'(2) = \left.\dfrac{72}{(2t+3)^2}\right\vert_{t=2}$ $= \dfrac{72}{49} = 1.469$ meters/sec^2	$2\begin{cases} 1: \text{ acceleration for Runner } A \\ 1: \text{ acceleration for Runner } B \end{cases}$
(c) Runner A: distance $= \dfrac{1}{2}(3)(10) + 7(10) = 85$ meters Runner B: distance $= \displaystyle\int_0^{10} \frac{24t}{2t+3}\,dt = 83.336$ meters	$4\begin{cases} 2: \text{ distance for Runner } A \\ \quad 1: \text{ method} \\ \quad 1: \text{ answer} \\ 2: \text{ distance for Runner } B \\ \quad 1: \text{ integral} \\ \quad 1: \text{ answer} \end{cases}$
(units) meters/sec in part (a), meters/sec^2 in part (b), and meters in part (c), or equivalent.	1: units

The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.



(a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

(b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

(c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

(d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

(a) $x = -1$ $f'(x)$ changes from negative to positive at $x = -1$.	$2\begin{cases}1: \text{ answer}\\ 1: \text{ justification}\end{cases}$
(b) $x = -5$ $f'(x)$ changes from positive to negative at $x = -5$.	$2\begin{cases}1: \text{ answer}\\ 1: \text{ justification}\end{cases}$
(c) $f''(x)$ exists and f' is decreasing on the intervals $(-7,-3)$, $(2,3)$, and $(3,5)$	$2\begin{cases}1: \ (-7,-3)\\ 1: \ (2,3)\cup(3,5)\end{cases}$
(d) $x = 7$ The absolute maximum must occur at $x = -5$ or at an endpoint. $f(-5) > f(-7)$ because f is increasing on $(-7,-5)$ The graph of f' shows that the magnitude of the negative change in f from $x = -5$ to $x = -1$ is smaller than the positive change in f from $x = -1$ to $x = 7$. Therefore the net change in f is positive from $x = -5$ to $x = 7$, and $f(7) > f(-5)$. So $f(7)$ is the absolute maximum.	$3\begin{cases}1: \text{ answer}\\ 1: \text{ identifies } x = -5 \text{ and } x = 7 \\ \quad\ \text{as candidates}\\ \quad\ \text{– or –}\\ \quad\ \text{indicates that the graph of } f\\ \quad\ \text{increases, decreases, then increases}\\ 1: \text{ justifies } f(7) > f(-5)\end{cases}$

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

(a) Method 1: $\int_0^3 \sqrt{t+1}\,dt = \frac{2}{3}(t+1)^{3/2}\Big|_0^3 = \frac{14}{3}$

– or –

Method 2: $L(t)$ = gallons leaked in first t minutes

$$\frac{dL}{dt} = \sqrt{t+1};\quad L(t) = \frac{2}{3}(t+1)^{3/2} + C$$

$$L(0) = 0;\quad C = -\frac{2}{3}$$

$$L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3};\quad L(3) = \frac{14}{3}$$

Method 1:

$$3\begin{cases} 2 : \text{definite integral} \\ \quad 1 : \text{limits} \\ \quad 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

– or –

Method 2:

$$3\begin{cases} 1 : \text{antiderivative with } C \\ 1 : \text{solves for } C \text{ using } L(0) = 0 \\ 1 : \text{answer} \end{cases}$$

(b) $30 + 8 \cdot 3 - \frac{14}{3} = \frac{148}{3}$

1 : answer

(c) Method 1:

$$A(t) = 30 + \int_0^t \left(8 - \sqrt{x+1}\right)dx$$
$$= 30 + 8t - \int_0^t \sqrt{x+1}\,dx$$

– or –

Method 2:

$$\frac{dA}{dt} = 8 - \sqrt{t+1}$$

$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + C$$

$$30 = 8(0) - \frac{2}{3}(0+1)^{3/2} + C;\quad C = \frac{92}{3}$$

$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + \frac{92}{3}$$

Method 1:

$$2\begin{cases} 1 : 30 + 8t \\ 1 : -\int_0^t \sqrt{x+1}\,dx \end{cases}$$

– or –

Method 2:

$$2\begin{cases} 1 : \text{antiderivative with } C \\ 1 : \text{answer} \end{cases}$$

(d) $A'(t) = 8 - \sqrt{t+1} = 0$ when $t = 63$.

$A'(t)$ is positive for $0 < t < 63$ and negative for $63 < t < 120$. Therefore there is a maximum at $t = 63$.

$$3\begin{cases} 1 : \text{sets } A'(t) = 0 \\ 1 : \text{solves for } t \\ 1 : \text{justification} \end{cases}$$

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

(a) $y^2 + 2xy\frac{dy}{dx} - 3x^2y - x^3\frac{dy}{dx} = 0$

$$\frac{dy}{dx}\left(2xy - x^3\right) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

$$2\begin{cases} 1: \text{implicit differentiation} \\ 1: \text{verifies expression for } \frac{dy}{dx} \end{cases}$$

(b) When $x = 1$, $y^2 - y = 6$

$$y^2 - y - 6 = 0$$
$$(y - 3)(y + 2) = 0$$
$$y = 3,\ y = -2$$

At $(1, 3)$, $\frac{dy}{dx} = \frac{9 - 9}{6 - 1} = 0$

Tangent line equation is $y = 3$

At $(1, -2)$, $\frac{dy}{dx} = \frac{-6 - 4}{-4 - 1} = \frac{-10}{-5} = 2$

Tangent line equation is $y + 2 = 2(x - 1)$

$$4\begin{cases} 1:\ y^2 - y = 6 \\ 1:\ \text{solves for } y \\ 2:\ \text{tangent lines} \end{cases}$$

Note: 0/4 if not solving an equation of the form $y^2 - y = k$

(c) Tangent line is vertical when $2xy - x^3 = 0$

$x\left(2y - x^2\right) = 0$ gives $x = 0$ or $y = \frac{1}{2}x^2$

There is no point on the curve with x-coordinate 0.

When $y = \frac{1}{2}x^2$, $\frac{1}{4}x^5 - \frac{1}{2}x^5 = 6$

$$-\frac{1}{4}x^5 = 6$$
$$x = \sqrt[5]{-24}$$

$$3\begin{cases} 1:\ \text{sets denominator of } \frac{dy}{dx} \text{ equal to } 0 \\ 1:\ \text{substitutes } y = \frac{1}{2}x^2 \text{ or } x = \pm\sqrt{2y} \\ \quad\ \text{into the equation for the curve} \\ 1:\ \text{solves for } x\text{-coordinate} \end{cases}$$

Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

(a) Find a solution $y = f(x)$ to the differential equation satisfying $f(0) = \frac{1}{2}$.

(b) Find the domain and range of the function f found in part (a).

(a) $e^{2y}\,dy = 3x^2\,dx$

$$\frac{1}{2}e^{2y} = x^3 + C_1$$

$$e^{2y} = 2x^3 + C$$

$$y = \frac{1}{2}\ln\left(2x^3 + C\right)$$

$$\frac{1}{2} = \frac{1}{2}\ln(0 + C); \quad C = e$$

$$y = \frac{1}{2}\ln(2x^3 + e)$$

6 {
- 1 : separates variables
- 1 : antiderivative of dy term
- 1 : antiderivative of dx term
- 1 : constant of integration
- 1 : uses initial condition $f(0) = \frac{1}{2}$
- 1 : solves for y

Note: 0/1 if y not a logarithmic function of x

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

(b) Domain: $2x^3 + e > 0$

$$x^3 > -\frac{1}{2}e$$

$$x > \left(-\frac{1}{2}e\right)^{1/3} = -\left(\frac{1}{2}e\right)^{1/3}$$

Range: $-\infty < y < \infty$

3 {
- 1 : $2x^3 + e > 0$
- 1 : domain

  Note: 0/1 if 0 is not in the domain
- 1 : range

Note: 0/3 if y is not a logarithmic function of x.

The Taylor series about $x = 5$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x = 5$ is given by $f^{(n)}(5) = \dfrac{(-1)^n n!}{2^n (n+2)}$, and $f(5) = \dfrac{1}{2}$.

(a) Write the third-degree Taylor polynomial for f about $x = 5$.

(b) Find the radius of convergence of the Taylor series for f about $x = 5$.

(c) Show that the sixth-degree Taylor polynomial for f about $x = 5$ approximates $f(6)$ with error less than $\dfrac{1}{1000}$.

(a) $f'(5) = \dfrac{-1!}{2(3)}$, $f''(5) = \dfrac{2!}{4(4)}$, $f'''(5) = \dfrac{-3!}{8(5)}$

$$P_3(f,5)(x) = \frac{1}{2} - \frac{1}{6}(x-5) + \frac{1}{16}(x-5)^2 - \frac{1}{40}(x-5)^3$$

3 : $P_3(f,5)(x)$

<–1> each error or missing term

Note: <–1> max for improper use of extra terms, equality or +…

(b) $a_n = \dfrac{f^{(n)}(5)}{n!} = \dfrac{(-1)^n}{2^n (n+2)}$

$$\lim_{n\to\infty}\left|\frac{\dfrac{(-1)^{n+1}(x-5)^{n+1}}{2^{n+1}(n+3)}}{\dfrac{(-1)^n(x-5)^n}{2^n(n+2)}}\right| = \lim_{n\to\infty}\frac{1}{2}\left(\frac{n+2}{n+3}\right)|x-5|$$

$$= \frac{|x-5|}{2} < 1$$

The radius of convergence is 2.

4 $\begin{cases} 1: \text{ general term} \\ 1: \text{ sets up ratio test} \\ 1: \text{ computes the limit} \\ 1: \text{ applies ratio test to get radius of convergence} \end{cases}$

(c) The Taylor series about $x = 5$ for the function f, when evaluated at $x = 6$, is an alternating series with absolute value of terms decreasing to 0. The error in approximating $f(6)$ with the 6th degree Taylor polynomial at $x = 6$ is less than the first omitted term in the series.

$$|f(6) - P_6(f,5)(6)| \le \frac{1}{2^7(9)} = \frac{1}{1152} < \frac{1}{1000}$$

2 $\begin{cases} 1: \text{ error bound} < \dfrac{1}{1000} \\ 1: \text{ refers to an alternating series and indicates the error bound is found from the next term} \end{cases}$

BC–4

A moving particle has position $(x(t), y(t))$ at time t. The position of the particle at time $t = 1$ is (2,6) and the velocity vector at any time $t > 0$ is given by $\left(1 - \frac{1}{t^2}, 2 + \frac{1}{t^2}\right)$.

(a) Find the acceleration vector at time $t = 3$.

(b) Find the position of the particle at time $t = 3$.

(c) For what time $t > 0$ does the line tangent to the path of the particle at $(x(t), y(t))$ have a slope of 8?

(d) The particle approaches a line as $t \to \infty$. Find the slope of this line. Show the work that leads to your conclusion.

(a) acceleration vector $= (x''(t), y''(t)) = \left(\frac{2}{t^3}, -\frac{2}{t^3}\right)$

$$(x''(3), y''(3)) = \left(\frac{2}{27}, -\frac{2}{27}\right)$$

$$2\left\{\begin{array}{l}1:\ \text{components of acceleration vector}\\ \quad\ \ \text{as a function of } t\\ 1:\ \text{acceleration vector at } t = 3\end{array}\right.$$

(b) $(x(t), y(t)) = \left(t + \frac{1}{t} + C_1,\ 2t - \frac{1}{t} + C_2\right)$

$$(2,6) = (x(1), y(1)) = (2 + C_1, 1 + C_2)$$

$$C_1 = 0,\ C_2 = 5$$

$$(x(3), y(3)) = \left(3 + \frac{1}{3}, 6 - \frac{1}{3} + 5\right) = \left(\frac{10}{3}, \frac{32}{3}\right)$$

$$3\left\{\begin{array}{l}1:\ \text{antidifferentiation}\\ 1:\ \text{uses initial condition at } t = 1\\ 1:\ \text{position at } t = 3\end{array}\right.$$

Note: max 1/3 [1–0–0] if no constants of integration

(c) $\frac{dy}{dx} = \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = 8$

$$2 + \frac{1}{t^2} = 8\left(1 - \frac{1}{t^2}\right);\quad t^2 = \frac{9}{6}$$

$$t = \sqrt{\frac{3}{2}}$$

$$2\left\{\begin{array}{l}1:\ \frac{dy}{dx} = 8 \text{ as equation in } t\\ 1:\ \text{solution for } t\end{array}\right.$$

(d) $\lim\limits_{t\to\infty} \frac{dy}{dx} = \lim\limits_{t\to\infty} \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = 2$

– or –

$$\lim_{t\to\infty} \frac{y(t)}{x(t)} = \lim_{t\to\infty} \frac{2t - \frac{1}{t} + 5}{t + \frac{1}{t}} = 2$$

$$2\left\{\begin{array}{l}1:\ \text{considers limit of } \frac{dy}{dx} \text{ or } \frac{y(t)}{x(t)}\\ 1:\ \text{answer}\end{array}\right.$$

Note: 0/2 if no consideration of limit

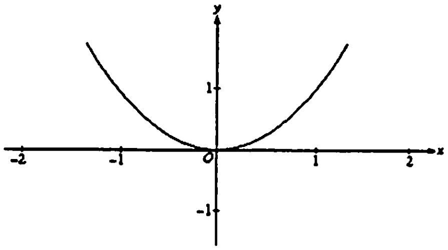
Consider the differential equation given by $\frac{dy}{dx} = x(y-1)^2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.

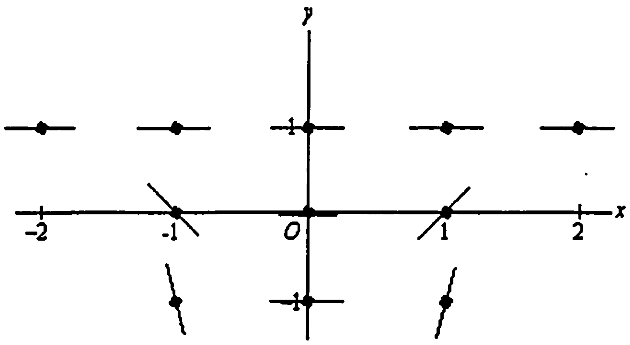
(b) Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -1$.

(d) Find the range of the solution found in part (c).



(a)



2 {
- 1 : zero slope at 7 points with $y = 1$ and $x = 0$
- 1 : negative slope at $(-1, 0)$ and $(-1, -1)$ positive slope at $(1, 0)$ and $(1, -1)$ steeper slope at $y = -1$ than $y = 0$

(b) The graph does not have slope 0 where $y = 1$.

– or –

The slope field shown suggests that solutions are asymptotic to $y = 1$ from below, but the graph does not exhibit this behavior.

1 : reason

(c) $\frac{1}{(y-1)^2}\,dy = x\,dx$

$$-\frac{1}{y-1} = \frac{1}{2}x^2 + C$$

$$\frac{1}{2} = 0 + C; \quad C = \frac{1}{2}$$

$$-\frac{1}{y-1} = \frac{1}{2}\left(x^2+1\right); \quad y = 1 - \frac{2}{x^2+1}$$

5 {
- 1 : separates variables
- 1 : antiderivatives
- 1 : constant of integration
- 1 : uses initial condition $f(0) = -1$
- 1 : solves for y
  - 0/1 if y is linear

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

(d) range is $-1 \le y < 1$

1 : answer

0/1 if –1 not in range

Pg 175

Subject: Re: Interval Notation
Date: 05/07 5:02 PM
Received: 05/07 8:51 PM
From: LnMcmullin@aol.com
Reply-To: Post AP Group, ap-calc@ets.org
To: Multiple recipients of list, ap-calc@ets.org

In a message dated 5/7/99 13:47:39 Eastern Daylight Time, Gregali@aol.com writes:

<< In identifying intervals in which a function is increasing or decreasing, should a student use open brackets or closed brackets, or is either one acceptable?

The same question holds for identifying intervals for concave up or concave down.

I've heard different opinions on this. Will someone please clarify, and provide an explanation, if possible? >>

Alison,

Either is acceptable. My own opinion is that such intervals for a continuous function are always closed. But so many book do not emphasize the distinction that on the AP exam either is considered okay.

The confusion usually arises because students often think that a function is increasing ONLY if it's derivative is positive. In fact a function is increasing on an interval if and only if, for all x1 and x2 in the interval if x1 < x2 then f(x1)<f(x2). (Definition of increasing). Notice there is no mention of the derivative.

A theorem says that if derivative is positive, then increasing; but its converse is FALSE.

Example: x^3 is increasing everywhere (including x = 0) by definition.

If a single point, say (x1,f(x1)), where the derivative is zero if f(x1)is less than all the points to its right then it is part of the interval where the function increases.

And likewise for decreasing and concavity.

Hope this helps.

Lin McMullin
Ballston Spa, NY

"increasing" involves an interval.
"... from x = a to x = b f is increasing ..."

critical value is at ~~x~~ a POINT.

$f(x) = x^3$

$f'(x) = 3x^2$

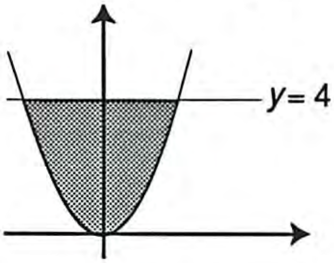
f is increasing

See pg 175
even though
for a single moment, at x=0
the ~~ta~~ tangent line to f
when flat.

1999

1. A particle moves along the y-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \geq 0$.
   - a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?
   - b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?
   - c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.
   - d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

2. The shaded region, R , is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure.
   - a) Find the area of R .
   - b) Find the volume of the solid generated by revolving R about the x-axis.
   - c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.



3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t . The table shows the rate as measured every 3 hours for a 24-hour period.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

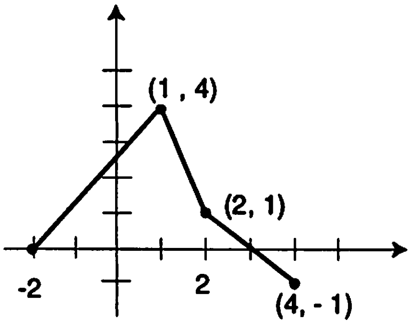
   - a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\, dt$. Using correct units, explain the meaning of your answer in terms of water flow.
   - b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.
   - c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

4. Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.
   - a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.
   - b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.
   - d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

I skipped "c"

c) Given that $g(0)=4$, write an equation to the line tangent to graph of g at the point where x=0

5. The graph of the function f, consisting of three line segments, is given. Let $g(x) = \int_1^x f(t)\, dt$.



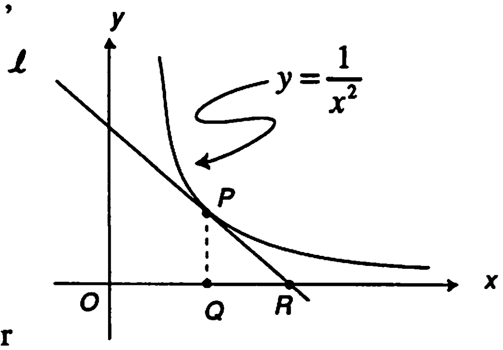
a) Compute $g(4)$ and $g(-2)$.

b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

6. In the figure, line ℓ is tangent to the graph of $y = \frac{1}{x^2}$ at point P, with coordinates $\left(w, \frac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x-axis at point R, with coordinates $(k, 0)$.



a) Find the value of k when $w = 3$.

b) For all $w > 0$, find k in terms of w.

c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?

d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of ΔPQR with respect to time? Determine whether the area is increasing or decreasing at this instant.

Student Performance Q&A:
1999 AP® Calculus AB/BC Section II

The following comments are provided by Bernard Madison (University of Arkansas-Fayetteville), Chief Faculty Consultant for AP Calculus, regarding the 1999 free-response questions for AP Calculus AB/BC. *They are intended to assist AP workshop consultants as they develop training sessions to help teachers better prepare their students for the AP Exams.* They give an overview of each question and its performance, including typical student errors. Some general comments regarding the skills and content that students frequently have the most problems with are included. Consultants are encouraged to use their expertise to create strategies for teachers to improve performance in specific areas.

AB Question 1:

The purpose of this question on particle motion was to test understanding of the relationships among position, distance traveled, velocity, speed, and acceleration. The mean score was 4.58, indicating a reasonably strong performance overall. The scores discriminated well at all four grade cuts, but best at the 1/2 and 2/3 cuts.

Most students gained the 3 points in parts (a) and (b) which asked for interpretation of directed changes in position and velocity in terms of signs of computed velocity and acceleration. Many students lost points in part (c) because they were unable to find the correct antiderivative of the given $v(t) = t\sin\left(t^2\right)$, and in part (d) because they could not handle the change in sign of $v(t)$ to compute distance traveled. Also, as has been the case in the past with questions involving trigonometric functions, some students did the computations with their calculators in degree mode.

AB/BC Question 2:

This question asked in parts (a) and (b) for computation of the area and the volume of the solid of revolution (about the x-axis) of the symmetric region bounded by the graphs of $y = 4$ and $y = x^2$. These two parts are very common applications of the definite integral and students who lost points usually did so by treating only half the symmetric region or omitting the factor of π in the volume. The unusual twist came in part (c) where the student needed to write an integral giving the volume of the solid generated by revolving the region about the line $y = k$ and setting this integral equal to the answer in part (b). Many students were not able to produce the integrand of the difference of the squares of the inner and outer radii.

The mean score was 4.37 for the AB students and 5.65 for the BC students, indicating a reasonably strong overall performance. As with question AB1, the scores discriminated well at all four grade cuts for both AB and BC, although not as well as AB1 at the 3/4 and 4/5 cuts for the AB population.

AB/BC Question 3:

The first part (a) of this three-part question asked for a Riemann sum approximation and an interpretation (as total flow) of the integral of a differentiable rate of flow of water given at 3-hour intervals in a table of values for a 24-hour period. The second part (b) asked students to explain why or why not the derivative of the rate function was zero at any time during this 24-hour period. Finally, the students were asked in (c) to assume that the rate is approximated by a given quadratic polynomial and compute an average rate of flow. Units were asked for in parts (a) and (c). One of the nine points was awarded for having correct units in both parts.

The AB mean score was 3.52 and the BC mean was 4.99, indicating a reasonable overall performance. The scores discriminated well at all four grade cuts on both AB and BC, and about equally well at all eight cuts.

Students had the most trouble in part (b) giving a reason why the derivative had to be zero somewhere in the interval from 0 to 24. This follows from the Mean Value Theorem (or Rolle's Theorem), but students gave arguments that contained fragments of the proof of these theorems. In part (c) students confused the average value with the average rate of change. The scoring of the point for units was complicated by students giving total flow for the given 24-hour period in part (a) in gallons per day or gallons/day rather than gallons. After much deliberation, in the interest of consistent grading, gallons/day was considered as incorrect units while gallons per day was read as total gallons during the day.

AB Question 4:

The intent of this question was to test the understanding of tangent lines, local extrema, and inflection points for functions that are only partially described. The first two parts, (a) and (b), asked for a tangent line to the graph of f at $x = 0$ and whether or not one could conclude that there was a point of inflection at $x = 0$. The only information given about f was that $f(0) = 2$, $f'(0) = -3$, $f''(0) = 0$, and that f had a continuous second derivative. Many students incorrectly answered yes to part (b) believing that $f''(0) = .0$ was sufficient to guarantee a point of inflection. In parts (c) and (d), students were asked about the function g defined by $g(x) = e^{-2x}(3f(x) + 2f'(x))$. In (c) the student was asked for a tangent line to the graph of g at $x = 0$ given that $g(0) = 4$. Finally, in (d) the student was asked whether or not g had a local maximum at $x = 0$. To answer, the student needed to compute $g'(0)$ and $g''(0)$, which many did not do. Further, many students' explanations referred to functions with the indefinite pronoun "it" which made their justifying arguments incomplete.

The mean score was 2.92, indicating a weak performance. However, the scores discriminated well at all four grade cuts.

AB/BC Question 5:

The intent of this question was to test understanding of the Fundamental Theorem of Calculus and to use that understanding to optimize a function defined by an integral and to analyze the points of inflection of the graph of that function.

The mean score was 1.54 for AB students and 3.26 for BC students, indicating a reasonable performance by the BC students and a very poor performance by the AB students. The latter is surprising and disappointing since questions of this type have appeared twice before on the AB exam in recent years. However, the scores did discriminate fairly well at the upper two grade cuts for both AB and BC.

In part (a) the common error was finding $g(-2) = 6$ rather than the correct -6.

Students had the most trouble with parts (c) and (d). In part (c) students were asked to find and justify the absolute minimum of the integrally defined function. This requires an argument that is both local, showing that the candidate value is a local minimum, and global, showing that the candidate value is the absolute minimum. In this case, the most apparent method is to compare the values of the function at the endpoints and the one interior critical point.

In part (d) students were asked to argue whether there were or were not points of inflection where $x = 1$ or where $x = 2$. Even when students were able to see what needed to be shown (i.e., that the second derivative did or did not change sign), many were not able to correctly state this or clearly identify which functions were meant.

The presence of the calculator may tempt some students to either curve fit the piecewise linear graph or at least write equations for the line segments and antidifferentiate analytically or with the calculator, producing a graph of the integrally defined function. These approaches are unwise and are likely to lead to a poor score.

AB Question 6:

The intent of this question was to have the student produce mathematical models and compute related rates of change of quantities associated with the tangent line to the graph of $y = \frac{1}{x^2}$.

First, the student is asked to find the x-intercept of the tangent line to the graph in part (a) when $x = 3$ and more generally in part (b) when $x = w$. In part (c) the rate of change of w is given as 7, and the student is asked to compute the rate of change of the x-intercept. Finally, part (d), the main thrust of the question, asks for a rate of change of the area of the right triangle with two vertices on the x-axis and the third at the point $\left(w, \frac{1}{w^2}\right)$ on the curve. Students had difficulty with the algebra in parts (b) and (d), and with the area model in part (d). Some of the difficulty was due to the several different quantities denoted by different letters in the question. Evidently, reading about a situation, even though it is a mathematically described one, and translating this into a mathematical model for the quantity needed (here the area of a right triangle) still poses difficulty for students.

The related rates exercises are made more difficult by students not simplifying algebraic expressions.

The mean score was 1.99, indicating a poor performance. However, the scores did discriminate well at the upper two grade cuts.

BC Question 1:

The intent of this question was to test the student's understanding of parametrically defined curves. Using this understanding the students were asked to minimize the x-coordinate on the curve and to compute the speed and acceleration vector at the time the particle is on the y-axis.

Part (a) required the student to give a graph of the curve, presumably copying it from the calculator window. Part (b) asked for the minimal x-coordinate, which required taking a derivative. Mistakes in taking the derivative were numerous and caused subsequent problems. In part (c) the student needed to solve an equation, finding a decimal value for the solution. Since this solution was to be used to find the acceleration vector, requiring more derivatives, and the speed, mistakes in the solution caused some problems in following student work. Many mistakes indicated a weak understanding of speed.

Scoring was complicated by trying to determine how students arrived at an incorrect decimal answer.

The mean score was 4.88, indicating a strong overall performance. The scores discriminated equally well at all grade cuts, almost identical to the pattern of BC3.

BC Question 4:

The intent of this question was to test understanding of Taylor polynomials and series, including error bound. All BC examinations in recent years have had a problem on series, and student performances have generally been weak. The mean score on this question was 3.25, again relatively weak. Moreover, this was the weakest discriminator among the BC questions.

Common errors included giving the Taylor polynomial about some number other than 2, often about 0 (Maclaurin polynomial). The unusual part of this question was in part (b) where the students were asked to use the Lagrange error bound on the approximation of $f(1.5)$ to show that $f(1.5)$ could not be -5. Student performance was weaker here.

BC Question 6:

The intent of this question was to demonstrate two approximations of $f(3.1)$ given $f'(x)$ and $f(3)$ and then to use the Fundamental Theorem of Calculus to find $f(3.1)$. This question was split graded with parts (a) and (c) [approximation with tangent line and the Fundamental Theorem calculation for 5 points] contributing to the AB subscore grade and part (b) [approximation by Euler's method for 4 points] being BC-only material.

The mean score on parts (a) and (c) was 3.17 (out of 5) and on part (b) 1.19 (out of 4), for a total mean of 4.37. This indicates a reasonably strong student performance. The scores discriminated well and almost equally at all grade cuts.

In part (a) many students lost points by not giving the equation of the tangent line. In part (b) it was often difficult to determine whether or not Euler's method was being used. The most difficult point on the entire BC examination was the final point of (b) which asked that the student use f'' to explain why the Euler approximation of $f(3.1)$ was less than $f(3.1)$. Very few students answered this correctly.

In part (c) students would compute the given integral without connecting it to $f(3.1)$ via the Fundamental Theorem.

MEAN: 4.58

QUESTION # 1 / STANDARDS

AB–1

1999

1. A particle moves along the y-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \geq 0$.
   (a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?
   (b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?
   (c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.
   (d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

(a) $v(1.5) = 1.5\sin(1.5^2) = 1.167$ Up, because $v(1.5) > 0$	1: answer and reason
(b) $a(t) = v'(t) = \sin t^2 + 2t^2\cos t^2$ $a(1.5) = v'(1.5) = -2.048$ or -2.049 No; v is decreasing at 1.5 because $v'(1.5) < 0$	$2\begin{cases}1\text{: } a(1.5)\\ 1\text{: conclusion and reason}\end{cases}$
(c) $y(t) = \int v(t)\,dt$ $= \int t\sin t^2\,dt = -\dfrac{\cos t^2}{2} + C$ $y(0) = 3 = -\frac{1}{2} + C \implies C = \frac{7}{2}$ $y(t) = -\frac{1}{2}\cos t^2 + \frac{7}{2}$ $y(2) = -\frac{1}{2}\cos 4 + \frac{7}{2} = 3.826$ or 3.827	$3\begin{cases}1\text{: } y(t) = \int v(t)\,dt\\ 1\text{: } y(t) = -\frac{1}{2}\cos t^2 + C\\ 1\text{: } y(2)\end{cases}$
(d) distance $= \int_0^2 \lvert v(t)\rvert\,dt = 1.173$ or $v(t) = t\sin t^2 = 0$ $t = 0$ or $t = \sqrt{\pi} \approx 1.772$ $y(0) = 3$; $y(\sqrt{\pi}) = 4$; $y(2) = 3.826$ or 3.827 $[y(\sqrt{\pi}) - y(0)] + [y(\sqrt{\pi}) - y(2)]$ $= 1.173$ or 1.174	$3\begin{cases}1\text{: limits of 0 and 2 on an integral of } v(t) \text{ or } \lvert v(t)\rvert\\ \quad\text{or}\\ \quad\text{uses } y(0) \text{ and } y(2) \text{ to compute distance}\\ 1\text{: handles change of direction at student's turning point}\\ 1\text{: answer}\\ \quad 0/1 \text{ if incorrect turning point}\end{cases}$

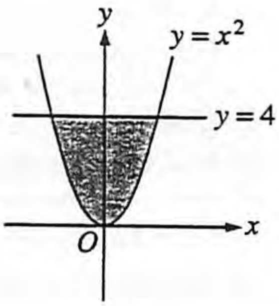
MEAN: AB 4.37
BC 5.65

QUESTION # 2 / STANDARDS

AB–2 / BC–2

1999

2. The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.
   (a) Find the area of R.
   (b) Find the volume of the solid generated by revolving R about the x–axis.
   (c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.



(a) $$\text{Area} = \int_{-2}^{2} (4 - x^2)\,dx = 2\int_{0}^{2} (4 - x^2)\,dx = 2\left[4x - \frac{x^3}{3}\right]_0^2 = \frac{32}{3} = 10.666 \text{ or } 10.667$$

$2 \begin{cases} 1\text{: integral} \\ 1\text{: answer} \end{cases}$

(b) $$\text{Volume} = \pi\int_{-2}^{2} \left(4^2 - (x^2)^2\right)dx = 2\pi\int_{0}^{2} (16 - x^4)\,dx = 2\pi\left[16x - \frac{x^5}{5}\right]_0^2 = \frac{256\pi}{5} = 160.849 \text{ or } 160.850$$

$3 \begin{cases} 1\text{: limits and constant} \\ 1\text{: integrand} \\ 1\text{: answer} \end{cases}$

(c) $$\pi\int_{-2}^{2} \left[(k - x^2)^2 - (k - 4)^2\right]dx = \frac{256\pi}{5}$$

$4 \begin{cases} 1\text{: limits and constant} \\ 2\text{: integrand} \\ \quad <-1> \text{ each error} \\ 1\text{: equation} \end{cases}$

MEAN: AB 3.52
BC 4.99

AB–3 / BC–3

1999

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.
   (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.
   (b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.
   (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

(a) $$\int_0^{24} R(t)\,dt \approx 6[R(3) + R(9) + R(15) + R(21)] = 6[10.4 + 11.2 + 11.3 + 10.2] = 258.6 \text{ gallons}$$

This is an approximation to the total flow in gallons of water from the pipe in the 24-hour period.

$$3\begin{cases} 1: R(3) + R(9) + R(15) + R(21) \\ 1: \text{answer} \\ 1: \text{explanation} \end{cases}$$

(b) Yes;

Since $R(0) = R(24) = 9.6$, the Mean Value Theorem guarantees that there is a t, $0 < t < 24$, such that $R'(t) = 0$.

$$2\begin{cases} 1: \text{answer} \\ 1: \text{MVT or equivalent} \end{cases}$$

(c) Average rate of flow

$\approx$ average value of $Q(t)$

$$= \frac{1}{24}\int_0^{24} \frac{1}{79}(768 + 23t - t^2)\,dt$$

$= 10.785$ gal/hr or 10.784 gal/hr

$$3\begin{cases} 1: \text{limits and average value constant} \\ 1: Q(t) \text{ as integrand} \\ 1: \text{answer} \end{cases}$$

(units) Gallons in part (a) and gallons/hr in part (c), or equivalent.

1: units

MEAN: AB 2.92

AB–4

1999

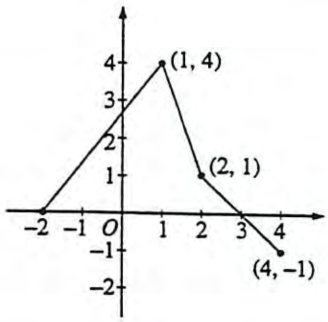
4. Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.
   (a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.
   (b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.
   (c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.
   (d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

(a) Slope at $x = 0$ is $f'(0) = -3$ At $x = 0$, $y = 2$ $y - 2 = -3(x - 0)$	1: equation
(b) No. Whether $f''(x)$ changes sign at $x = 0$ is unknown. The only given value of $f''(x)$ is $f''(0) = 0$.	2 $\begin{cases} \text{1: answer} \\ \text{1: explanation} \end{cases}$
(c) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ $g'(0) = e^0(3f(0) + 2f'(0))$ $= 3(2) + 2(-3) = 0$ $y - 4 = 0(x - 0)$ $y = 4$	2 $\begin{cases} \text{1: } g'(0) \\ \text{1: equation} \end{cases}$
(d) $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ $g''(x) = (-2e^{-2x})(3f(x) + 2f'(x)) + e^{-2x}(3f'(x) + 2f''(x))$ $= e^{-2x}(-6f(x) - f'(x) + 2f''(x))$ $g''(0) = e^0[(-6)(2) - (-3) + 2(0)] = -9$ Since $g'(0) = 0$ and $g''(0) < 0$, g does have a local maximum at $x = 0$. f'' test.	4 $\begin{cases} \text{2: verify derivative} \\ \quad \text{0/2 product or chain rule error} \\ \quad \text{<−1> algebra errors} \\ \text{1: } g'(0) = 0 \text{ and } g''(0) \\ \text{1: answer and reasoning} \end{cases}$

MEAN: AB 1.54
BC 3.26

AB–5 / BC–5

1999



5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)\,dt$.

   (a) Compute $g(4)$ and $g(-2)$.

   (b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.
   means deriv

   (c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

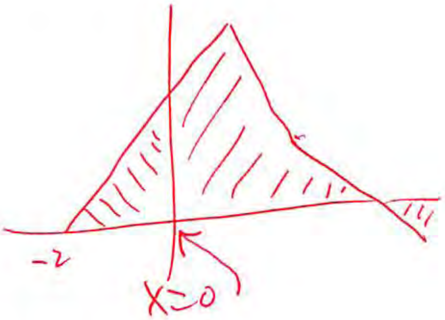
   (d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x–coordinates of points of inflection of the graph of g? Justify your answer.

(a) $g(4) = \int_1^4 f(t)\,dt = \frac{3}{2} + 1 + \frac{1}{2} - \frac{1}{2} = \frac{5}{2}$

$g(-2) = \int_1^{-2} f(t)\,dt = -\frac{1}{2}(12) = -6$

$2 \begin{cases} 1\text{: } g(4) \\ 1\text{: } g(-2) \end{cases}$

So $\int_0^x f(t)\,dt$.



(b) $g'(1) = f(1) = 4$

1: answer

(c) g is increasing on $[-2, 3]$ and decreasing on $[3, 4]$.

Therefore, g has absolute minimum at an endpoint of $[-2, 4]$.

Since $g(-2) = -6$ and $g(4) = \frac{5}{2}$,

the absolute minimum value is -6.

$3 \begin{cases} 1\text{: interior analysis} \\ 1\text{: endpoint analysis} \\ 1\text{: answer} \end{cases}$

(d) One; $x = 1$

On $(-2, 1)$, $g''(x) = f'(x) > 0$

On $(1, 2)$, $g''(x) = f'(x) < 0$

On $(2, 4)$, $g''(x) = f'(x) < 0$

Therefore $(1, g(1))$ is a point of inflection and $(2, g(2))$ is not.

$3 \begin{cases} 1\text{: choice of } x = 1 \text{ only} \\ 1\text{: show } (1, g(1)) \text{ is a point of inflection} \\ 1\text{: show } (2, g(2)) \text{ is not a point of inflection} \end{cases}$

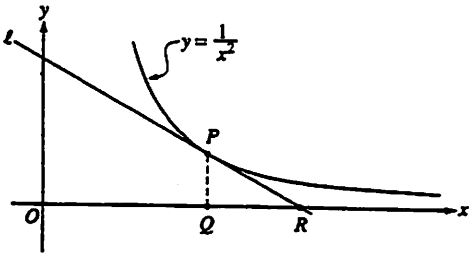
must ~~show~~ specifically state that 2, g(2) is not a pt. of infl.

MEAN: 1.99

AB–6

1999

6. In the figure above, line ℓ is tangent to the graph of $y = \frac{1}{x^2}$ at point P, with coordinates $\left(w, \frac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x-axis at the point R, with coordinates $(k, 0)$.



   (a) Find the value of k when $w = 3$.
   (b) For all $w > 0$, find k in terms of w.
   (c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?
   (d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of ΔPQR with respect to time? Determine whether the area is increasing or decreasing at this instant.

(a) $\frac{dy}{dx} = -\frac{2}{x^3}$; $\left.\frac{dy}{dx}\right|_{x=3} = -\frac{2}{27}$

Line ℓ through $\left(3, \frac{1}{9}\right)$ and $(k, 0)$ has slope $-\frac{2}{27}$.

Therefore, $\frac{0 - \frac{1}{9}}{k - 3} = -\frac{2}{27}$ or $0 - \frac{1}{9} = -\frac{2}{27}(k - 3)$

$k = \frac{9}{2}$

$$2\begin{cases} 1\text{: } \left.\frac{dy}{dx}\right|_{x=3} \\ 1\text{: answer} \end{cases}$$

(b) Line ℓ through $\left(w, \frac{1}{w^2}\right)$ and $(k, 0)$ has slope $-\frac{2}{w^3}$.

Therefore, $\frac{0 - \frac{1}{w^2}}{k - w} = -\frac{2}{w^3}$ or $0 - \frac{1}{w^2} = -\frac{2}{w^3}(k - w)$

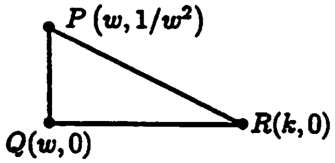
$k = \frac{3}{2}w$

$$2\begin{cases} 1\text{: equation relating } w \text{ and } k, \\ \quad\text{using slopes} \\ 1\text{: answer} \end{cases}$$

(c) $\frac{dk}{dt} = \frac{3}{2}\frac{dw}{dt} = \frac{3}{2} \cdot 7 = \frac{21}{2}$; $\left.\frac{dk}{dt}\right|_{w=5} = \frac{21}{2}$

1: answer using $\frac{dw}{dt} = 7$

(d)



$A = \frac{1}{2}(k - w)\frac{1}{w^2} = \frac{1}{2}\left(\frac{3}{2}w - w\right)\frac{1}{w^2} = \frac{1}{4w}$

$\frac{dA}{dt} = -\frac{1}{4w^2}\frac{dw}{dt}$

$\left.\frac{dA}{dt}\right|_{w=5} = -\frac{1}{100} \cdot 7 = -0.07$

Therefore, area is decreasing.

$$4\begin{cases} 1\text{: area in terms of } w \text{ and/or } k \\ 1\text{: } \frac{dA}{dt} \text{ implicitly} \\ 1\text{: } \left.\frac{dA}{dt}\right|_{w=5} \text{ using } \frac{dw}{dt} = 7 \\ 1\text{: conclusion} \end{cases}$$

Note: 0/4 if A constant

1999

① d 1.173

② c washers $\pi\int_0^2 (K-x^2)^2-(K-4)^2\,dx \overset{\text{set}}{=} \frac{256\pi}{5} = 160.850$

or

shells $2\cdot 2\pi\int_0^4 \sqrt{y}\,(K-y)\,dy \overset{\text{set}}{=} \frac{256\pi}{5}$

③ b Yes a point exists. $R(t)$ is cont & diff and $R(0)=R(24)$ so by Rolle's thm

④ b No there is not enough info. Just because $f''(0)=0$ Doesn't guarantee change in concavity.

⑤ d $x=1$ Justified.

⑥ c $\frac{dK}{dt} = \frac{21}{2}$

both Right	+
1 Right	✓
1 right of one tried	0
both tried	—
1 Nothing	=

$v(t) = t \sin t^2 \quad t \ge 0$

1999
①

③ a) $v(1.5) = 1.5 \sin (1.5)^2$ ① Sentence ① Answer

$= 1.167 > 0 \therefore$ moving up the y axis's
① Reason

④ b) $a(t) \overset{①}{=} d(v(t))$

$a(t) = \sin t^2 + 2t^2 \cos t^2$ ①

$a(1.5) \overset{①}{=} -2.049 < 0 \therefore$ the velocity ② is NOT increasing at $t = 1.5$ since $a(1.5) < 0$

④ c) $y(t) \overset{①}{=} \int v(t)\,dt$

$= \int t \sin t^2\,dt$ let $u = t^2$
$du = 2t\,dt$
$\frac{1}{2}du = t\,dt$

$\frac{1}{2}\int \sin u\,du$

$-\frac{1}{2}\cos u + C$

$y(t) = -\frac{1}{2}\cos t^2 + C$ ①

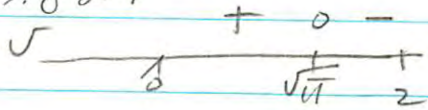
$y(0) = -\frac{1}{2} \cdot 1 + C \overset{set}{=} 3$

$C = \frac{7}{2}$

$\therefore y(t) = \frac{7}{2} - \frac{1}{2}\cos t^2$ ①

$y(2) = \frac{7}{2} - \frac{1}{2}\cos 4 = 3.827$ ①

④ d) $v = t \sin t^2 \overset{set}{=} 0$



$t = 0$ Reject or $t^2 = \pi$
$t = \sqrt{\pi}$

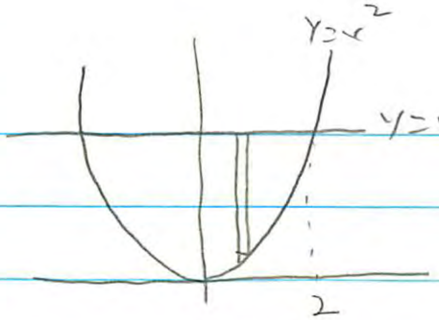
$\int_0^2 |t \sin t^2|\,dt \overset{①}{=} 1.173$ use the absolute value so we get total distance instead of net distance ①

② OR

$\int_0^{\sqrt{\pi}} t \sin t^2\,dt + \int_{\sqrt{\pi}}^{2} -t \sin t^2\,dt = 1.173$

1999

⑦



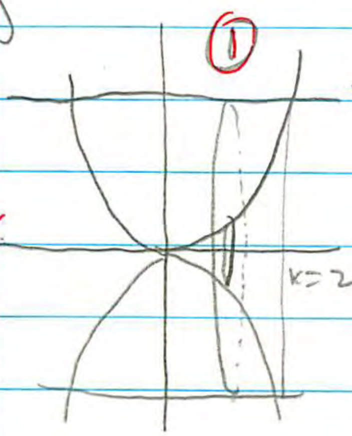
④ a) $2\int_0^2 4-x^2\,dx$ ②

②

$2\left(4x-\frac{x^3}{3}\right)\Big|_0^2 = 2\left(8-\frac{8}{3}\right) = \frac{32}{3}$

10.667

⑤ b)



①

shells

$2\pi\int_0^4 y\sqrt{y}\,dy$

$2\pi\int_0^4 y^{\frac{3}{2}}\,dy$

① "washers" $= \pi\int r_1^2 - r_2^2\,dx$

② $2\pi\int_0^2 4^2 - x^4\,dx =$

$2\pi\left(16x - \frac{x^5}{5}\right)\Big|_0^2 = 2\pi\left(32 - \frac{32}{5}\right)$ ①

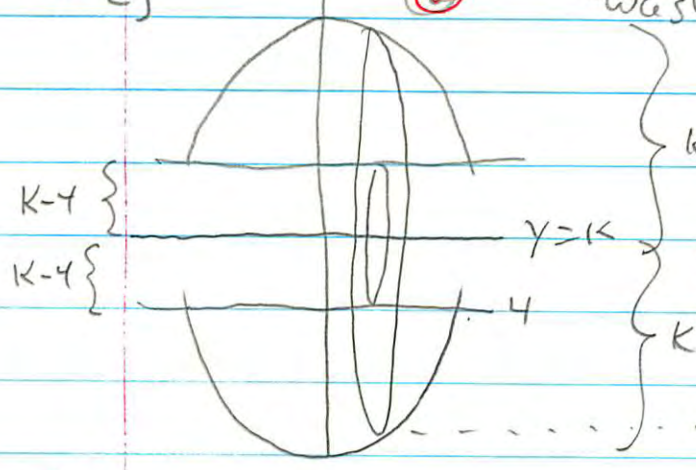
$2\pi\left(\frac{128}{5}\right) = \frac{256\pi}{5}$

160,850

51.2π

double on $\pi\int_{-2}^{2}$ etc

⑥ c)



②

① "washers"

$2\pi\int_0^2 \underset{①}{(k-x^2)^2} - \underset{①}{(k-4)^2}\,dx \overset{set}{=} ①$

$\frac{256\pi}{5}$

← this y-value . . . .

← minus this y value (4) for small radius

← minus this y value for large radius

shells:

$2\cdot 2\pi\int_0^4 \sqrt{y}\,(k-y)\,dy \overset{set}{=} \frac{256\pi}{5}$

1999

③

⑦ ⓐ

General Set up ②

0	9.6
3	10.4 ←
6	10.8
9	11.2 ←
12	11.4
15	11.3 ←
18	10.7
21	10.2 ←
24	9.6

with 6

$$6\overbrace{\left(10.4+11.2+11.3+10.2\right)}^{\text{Proper values ②}}$$

$$6(43.1)$$

$$258.6 \quad ①$$

In 24 hours approximately 258.6 gallons of water flowed through the pipe.

②

③ b) Yes R'(t) is The Rate of change in The gal/hr of The water. The Rate of change appears to be zero at T=12 since this is the max of The fctn R(t) and The max could occur at a zero of The der of the fctn.

⑤ c) $Q(t)=\frac{1}{79}(768+23t-t^2)$

②

$$\frac{1}{24}\int_0^{24} Q(t)\,dt \approx 10.785 \quad ①$$

Approximately 10.785 gallons of water per hour flowed through the pipe.

②

ⓑ R(t) is continuous and differentiable

$R(0)=9.6=R(24)$ ∴ by Rolle's thm There Exists X

$R(X)=0$.

Note had data been $R(0)=5$ $R(24)=9.6$ the argument could be That There exists c such That $R(c)=9.6$ by IVT then Rolle's Thm could have been applied.

1999
④

$f(0) = 2$
$f'(0) = -3$
$f''(0) = 0$

$$g'(x) = \overset{u}{e^{-2x}}\overset{v}{(3f(x) + 2f'(x))}$$

③ a) $f'(0) = -3 = m$ ①
PT $(0, f(0)) = (0, 2)$ ①

$y - y_1 = m(x - x_1)$
$y - 2 = -3(x - 0)$
$y = -3x + 2$ ①

② b) NO there is not enough information to determine if there is a PI at $x=0$. Just because $f''(0)=0$ isn't enough. The function must change concavity from up to down (or vice versa) If fact, f'' Doesn't even have to exist at the PI

based on reason only

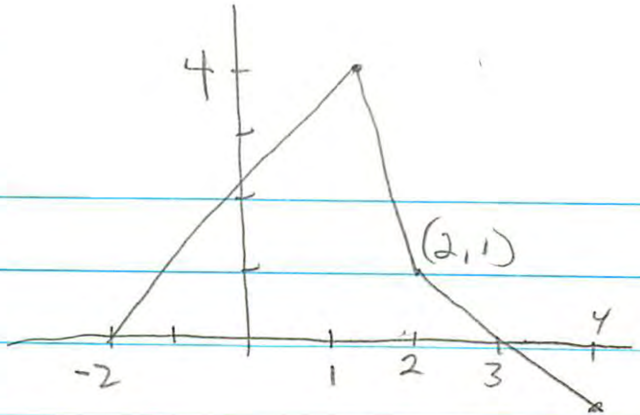
④ c) $g(0) = 4$
$m = g'(0) = e^0(3f(0) + 2f'(0))$
$1\ (3 \cdot 2 + 2(-3)$
$= 0$ ②
$\therefore$ equation $y = 4$ ②

⑥ d) from top of page, Product rule

$$g''(x) = (3f(x + 2f'(x))(-2e^{-2x}) + e^{-2x}[3f'(x) + 2f''(x)]$$
$$g''(x) = e^{-2x}[-6f(x) - 4f'(x) + 3f'(x) + 2f''(x)]$$
$$= e^{-2x}[-6f(x) - f'(x) + 2f''(x)]$$ ②
$$g''(0) = 1\ [-6(2) - (-3) + 2(0)]$$
$$= -9$$ ①

$g'(0) = 0$ (From Part c) so zero is critical value ①
$g''(0) < 0 \therefore (0, g(0))$ is a max ① f'' ~~test~~ ①

1999
⑤



$g(x) = \int_1^x f(\tau)\,d\tau$

⑤ a) $g(4) = \int_1^4 f(\tau)\,d\tau = \frac{1}{2}\cdot 1\cdot 3 + 1\cdot 1 + \frac{1}{2} - \frac{1}{2}$

recognition of concept ②

$\frac{5}{2}$ ①

$g(-2) = \int_1^{-2} f(\tau)\,d\tau$

$-\int_{-2}^{1} f(\tau)\,d\tau = -\left(\frac{1}{2}\cdot 3\cdot 4\right)$

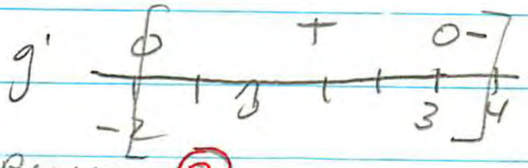
-6 ② including the negative.

③ b) $g(x) = \int_1^x f(\tau)\,d\tau$ concept ②

$g'(x) = f(x)\Big|_{x=1} = 4$ ① from Graph

④ c) $g'(x) = f(x) \overset{\text{set}}{=} 0$

$x \in \{-2, 3\}$

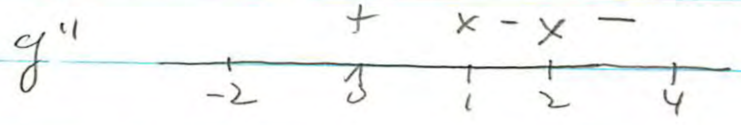


Reasons ②

$g(-2)$ is a min END PT INC from it $(-2, -6)$
$g(4)$ is a min END PT DEC to it. $(4, \frac{5}{2})$ } from part a

Abs min is -6 when $x = -2$ ②

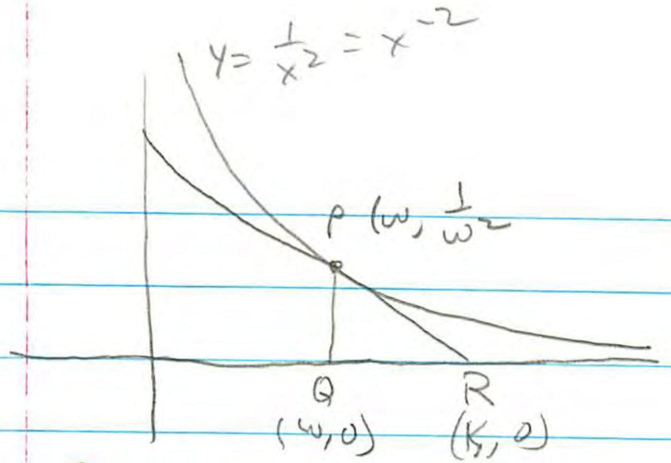
③ D)



① Note: g'' is the Slope of the graph of g'.

The only PI occurs at $x=1$ because g' changes from plus to minus so g changes from concave up to concave down. ②

1999
⑥



$y - y_1 = m(x - x_1)$

$y - 0 = \frac{-2}{27}(x - k)$

$y = \frac{-2}{27}x + \frac{2}{27}k$

⑤ ③ a) $(3, \frac{1}{9}), (k, 0)$ $y' = -\frac{2}{x^3}\Big|_{x=3} = \frac{-2}{27}$ ①

$\frac{\frac{1}{9} - 0}{3 - k} \overset{set}{=} \frac{-2}{27}$ ①

$3 = 2k - 6$

$2k = 9$

$k = \frac{9}{2}$ ①

④ ③ b) $\frac{\frac{1}{w^2} - 0}{w - k} \overset{set}{=} \frac{-2}{w^3}$

$w = 2k - 2w$ ③

$3w = 2k$

$k = \frac{3}{2}w$ ④

③ c) $\frac{dw}{dt} = 7$ $\frac{dk}{dt}\Big|_{w=5} = ?$ ①

$k = \frac{3}{2}w$

$\frac{dk}{dt} = \frac{3}{2} \cdot \frac{dw}{dt}$ ①

$= \frac{3}{2} \cdot 7$

$= \frac{21}{2}$ ①

⑥ d) $\frac{dw}{dt} = 7$ $A = \frac{1}{2}(k - w)\frac{1}{w^2}$ ①

① $\frac{dA}{dt}\Big|_{w=5} = ?$

$= \frac{1}{2}(\frac{3}{2}w - w)\frac{1}{w^2}$

$= \frac{1}{2}(\frac{1}{2}w)\frac{1}{w^2}$

$= \frac{1}{4w}$ ①

$\frac{dA}{dt} = -\frac{1}{4w^2}\frac{dw}{dt}$ ①

$= -\frac{1}{4 \cdot 25} \cdot 7$

$= -\frac{7}{100}$ ①

The area is decreasing at the rate of $\frac{7}{100}$ sq units/sec. ①

1998

① b $h = \sqrt[3]{16} = 2.520$

② b only min is $\frac{-1}{e}$ ∴ it is absolute $-.368$

③ d "the car traveled about 2530 feet."

④ c $f(x) = \sqrt{x^3 + x + 14}$ AND a statement that $y^2 = x^3 + x + c$ leads to ± answer but gives $f(1) > 0$.

⑤ D \$5.10 total cost to cool home for 24 hrs.

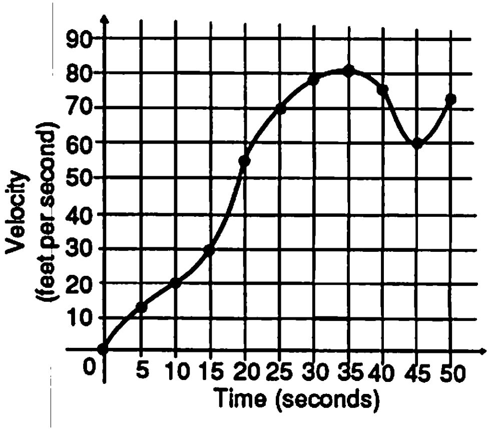
⑥ B $y = .165$ only tangent line w/ slope zero.

Calculus AB **98**

1. Let $\boldsymbol{R}$ be the region bounded by the x-axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.
   a) Find the area of the region $\boldsymbol{R}$.
   b) Find the value of h such that the vertical line $x = h$ divides the region $\boldsymbol{R}$ into two regions of equal area.
   c) Find the volume of the solid generated with $\boldsymbol{R}$ is revolved about the x-axis.
   d) The vertical line $x = k$ divides the region $\boldsymbol{R}$ into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k.

2. Let f be the function given by $f(x) = 2xe^{2x}$
   a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.
   b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.
   c) What is the range of f?
   d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown on the right. A table of values for $v(t)$, at 5 second intervals of time t, is shown as well.
   a) During what intervals of time is the acceleration of the car positive? Explain.
   b) Find the average acceleration of the car, in $\mathrm{ft/sec^2}$, over the interval $0 \le t \le 50$.
   c) Find one approximation for the acceleration of the car, in $\mathrm{ft/sec^2}$, at $t = 40$. Show the computation you used to arrive at your answer.
   d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

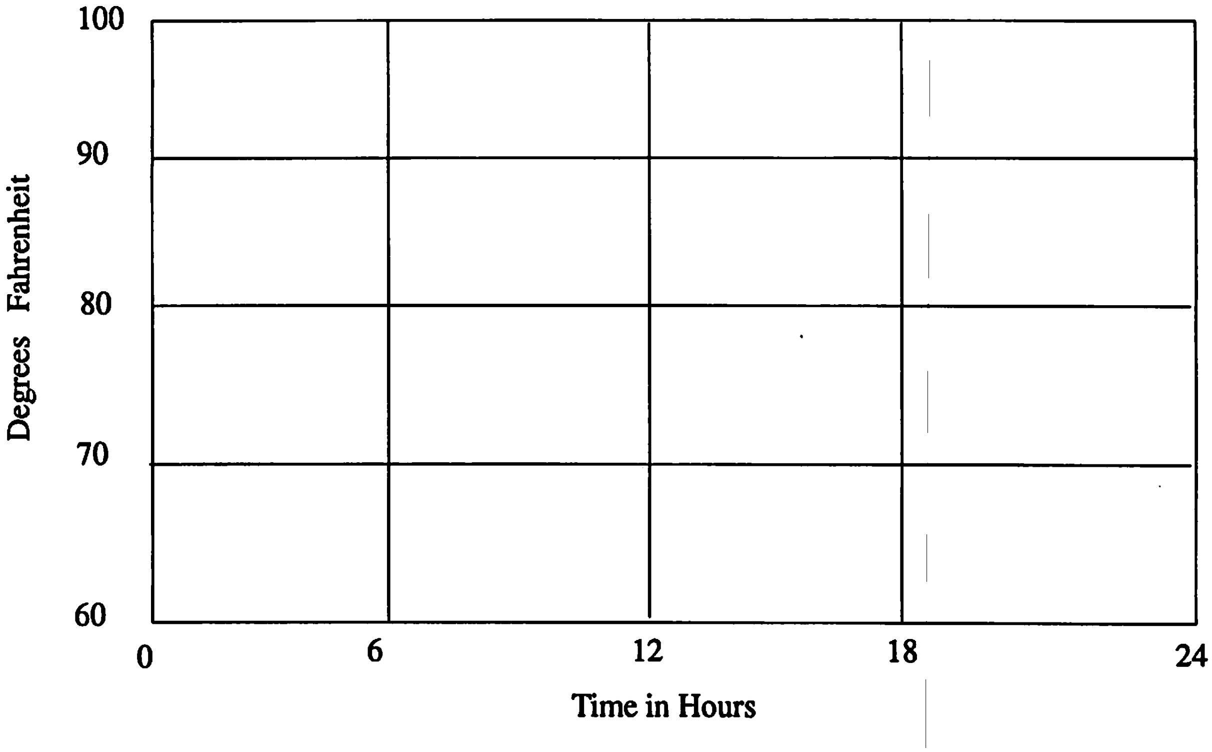


t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

4. Let f be a function with $f(1) = 4$ such that all points (x, y) on the graph of f the slope is given by: $\dfrac{3x^2 + 1}{2y}$.
   a) Find the slope of the graph of f at the point where $x = 1$.
   b) Write an equation for the line tangent to the graph of f at $x = 1$ an use it to approximate $f(1.2)$.
   c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2 + 1}{2y}$ with initial condition $f(1) = 4$.
   d) Use your solution from part (c) to find $f(1.2)$.

5. The temperature outside a house during a 24-hour period is given by $F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right),\ 0 \le t \le 24$ where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

a) Sketch the graph of $F(t)$ on a grid such as the following:



b) Find the average temperature, to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?

d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside Temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

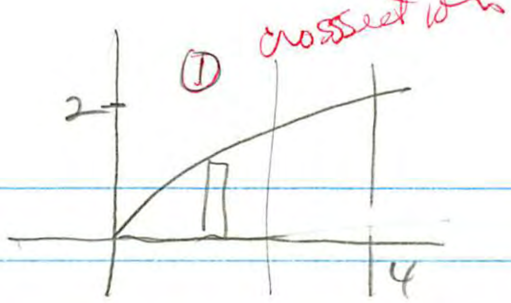
6. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

a) Show that $\dfrac{dy}{dx} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$.

b) Write an equation of each horizontal tangent line to the curve.

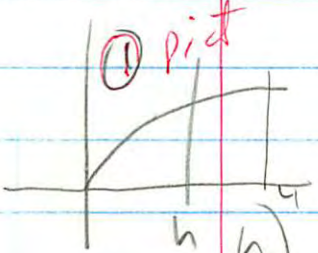
c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x- and y-coordinate of point P.

98
①



③ a) ① $\int_0^4 \sqrt{x}\,dx =$

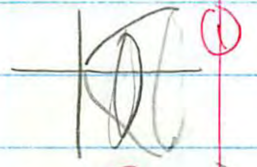
$\frac{2x^{\frac{3}{2}}}{3}\Big|_0^4 = \frac{2}{3}\cdot 8 = \frac{16}{3}$ ①



① b) ④ $\int_0^{h} \sqrt{x}\,dx \overset{\text{set}}{=} \frac{8}{3}$ ①

$\frac{2}{3}x^{\frac{3}{2}}\Big|_0^h = \frac{2}{3}h^{\frac{3}{2}} \overset{\text{set}}{=} \frac{8}{3}$

$\left(h^{\frac{3}{2}}\right)^{\frac{2}{3}} = (4)^{\frac{2}{3}}$

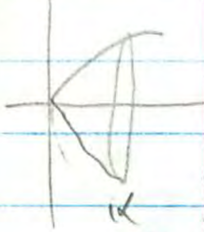


$h = \sqrt[3]{16}$ ① ≈ 2.520

④ c) ① "disks" $\pi\int r^2\,dx$ $2\sqrt[3]{2}$

① $\pi\int_0^4 x\,dx$

$\pi\frac{x^2}{2}\Big|_0^4 = 8\pi$ ① ≈ 25.133



D) ④ ① $\pi\int_0^K x\,dx \overset{\text{set}}{=} 4\pi$ ① ① (half of volume)

$\frac{x^2}{2}\Big|_0^K = 4$

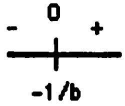
$K^2 = 8$ K > 0

$K = 2\sqrt{2}$ ① 2.828

2. d. Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b

Case 1 $b > 0$. Then limit analysis in part a is still valid with b in place of 2. Thus the absolute minimum occurs at the critical point of $f(x)$
Find $f'(x) = be^{bx} + b^2xe^{bx}$ $f'(x) = 0 \dashrightarrow be^{bx} + b^2xe^{bx} = 0 \dashrightarrow$

$be^{bx}(1 + bx) = 0 \dashrightarrow 1 + bx = 0 \dashrightarrow x = -\frac{1}{b}$ The f' chart would be



The absolute minimum would then happen at $x = -\frac{1}{b}$ and the absolute minimum would be $b\left(-\frac{1}{b}\right)e^{b(-1/b)} = -e^{-1} = -\frac{1}{e}$

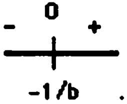
Case 2 $b < 0$

Analysis of part a would now be analyzing bxe^{bx} with b negative.

For positive x, $f(x) = -\left|\frac{bx}{e^{|b|x}}\right|$ which approaches $\boxed{0}$ as x approaches $-\infty$
For negative x, $f(x) = \left|bxe^{|bx|}\right|$ which gets large beyond any bound or $\boxed{+\infty}$

The $f'(x) = 0$ solution is the same as in Case 1 giving a value of $-\frac{1}{e}$

Thus, the function has no absolute maximum and has an absolute minimum at its critical point as indicated by the f' chart . Thus the solution is the same as in Case 1, or $\boxed{-\frac{1}{e}}$



Problem 3 Part 1

Return to Mr. Calculus

$$f(x) = \overset{u}{bx}\,\overset{v}{e^{bx}}$$

$$f'(x) = be^{bx} + b^2xe^{bx}$$

$$be^{bx}(1 + bx) \overset{set}{=} 0$$

~~If $b > 0$ then No Solution.~~

if $b > 0$ $\qquad - \;\; 0 \;\; +$

$x = -\frac{1}{b}$ $\qquad -\frac{1}{b} \quad 0$

if $b < 0$ $\qquad - \;\; 0 \;\; +$

$-\frac{1}{b} \quad 0$

9F
②

$f(x) = 2xe^{2x}$

③ a) $\lim\limits_{x \to -\infty} 2xe^{2x}$ let $a = -x$ ①

$\lim\limits_{a \to \infty} \frac{-2a}{e^{2a}}$ $\frac{-\infty}{\infty}$ $\lim\limits_{a \to \infty} 2xe^{2x} \to \infty$ No limit.

$\lim\limits_{a \to \infty} \frac{-2}{2e^{2a}} \to 0$ ① Reason ①

⑤ b) $f(x) = \overset{u}{2x}\overset{v}{e^{2x}}$ $f' \; \underset{-\frac{1}{2}}{\overset{- \;\; 0 \;\; +}{\longrightarrow}}$

① $f'(x) = 2e^{2x} + 4xe^{2x}$

$2e^{2x}(1 + 2x)$ min at $x = -\frac{1}{2}$ f' test ①

$f(-\frac{1}{2}) = 2(-\frac{1}{2})e^{2(-\frac{1}{2})}$ ①

$= -\frac{1}{e}$ ① The only (therefore absolute) min of f is $-\frac{1}{e}$. ①

② c) R_f $f(x) \geq -\frac{1}{e}$ Range is greater than or equal to Abs min!

5 ⑤ d) $f(x) = \overset{u}{bx}\overset{v}{e^{bx}}$

Large b $be^{bx}(1 + bx)$

If $b > 0$ + + = +

If $b < 0$ − − = +

③ { $f'(x) = be^{bx} + b^2xe^{bx}$

$be^{bx}(1 + bx) \overset{\text{set}}{=} 0$ $\underset{-\frac{1}{b}}{\overset{- \;\; 0 \;\; +}{\longrightarrow}}$ Still min. f' test.

$f(-\frac{1}{b}) = b(\frac{-1}{b})e^{b(\frac{-1}{b})}$

$= -\frac{1}{e}$ ① as in part b. Thus for all $b \neq 0$ The min is $-\frac{1}{e}$. ①

98

(3)

Reason ②

④ a) acceleration is when slope of velocity is positive. The slope is positive for $t \in (0, 35)$ ① and $t \in (45, 50)$ ①

④ b) Average ① acceleration is the average of the slopes...

take ten consecutive pairs of points and computing the slope gives

This method has style!

$$\frac{12}{5} + \frac{8}{5} + \frac{10}{5} + \frac{25}{5} + \frac{15}{5} + \frac{8}{5} + \frac{3}{5} - \frac{6}{5} - \frac{15}{5} + \frac{12}{5}$$

Average Value ②: $\frac{1}{b-a}\int_a^b f(x)\,dx$

$$\frac{1}{50-0}\int_0^{50} a(t)\,dt =$$

$$\frac{1}{50} v(t)\Big|_0^{50} = \frac{1}{50} \cdot 72 = \frac{36}{25}$$

$$\frac{81}{5} - \frac{21}{5} + \frac{12}{5}$$

$\frac{72}{5}$ total of 10 slopes

$$\frac{72}{5 \cdot 10} = \frac{36}{25}. \text{ ft/sec}^2 \text{ ①}$$

④ c) average at $t = 40$ is best approximated by chord from $(35, 80)$ and $(45, 60)$

also acceptable

$$\frac{v(45) - v(40)}{5} = -3$$

or

$$\frac{v(40) - v(35)}{5} = -\frac{6}{5}$$

or

$$\frac{v(35) - v(45)}{10} = -3$$

method Mason ②

$$\frac{81 - 60}{35 - 45} = \frac{21}{-10} = -2.1 \text{ ①}$$

Thus the acceleration was -2 i.e. the car was slowing at 2 ft/sec². ①

recog Neg.

③ d) $\int_0^{50} v(t)\,dt \approx 10\,[12 + 30 + 70 + 81 + 60]$ ①

$10\,[112 + 141]$

$10\,[253]$

2,530. in the 1st 50 seconds the car travels about 2,530 feet.
① ①

98
④

$f(1)=4 \qquad \frac{dy}{dx}=\frac{3x^2+1}{2y}$ Note: f is a fctn not a relation.

② a) $f'(1)=?$ $\quad \frac{dy}{dx}=\frac{3(1)+1}{8}$

$y\big|_{x=1}=f(1)=4$ ① $\quad \longrightarrow \quad = \frac{4}{8}=\frac{1}{2}$ ①

③ b) $(1,4) \quad m=\frac{1}{2} \qquad y-4=\frac{1}{2}(x-1)$

$y=\frac{1}{2}x+\frac{7}{2}$ ②

$y\big|_{1.2}=\frac{1}{2}\cdot\frac{6}{5}+\frac{35}{10}$

$=\frac{41}{10}=4.100$ ①

⑦ c) $\frac{dy}{dx}=\frac{3x^2+1}{2y}$

$\int 2y\,dy=\int 3x^2+1\,dx$ ①

$y^2=x^3+x+C$ ①

Makes it a fctn. ② $y=\sqrt{x^3+x+C}$ ① explain. Positive Root because y is either >0 or <0 and f(1) = 4 > 0.

$4=\sqrt{1+1+C}$

$16=2+C$

$c=14$ ①

$f(x)=\sqrt{x^3+x+14}$ ①

③ D) $f(1.2)=\sqrt{(1.2)^3+1.2+14}$

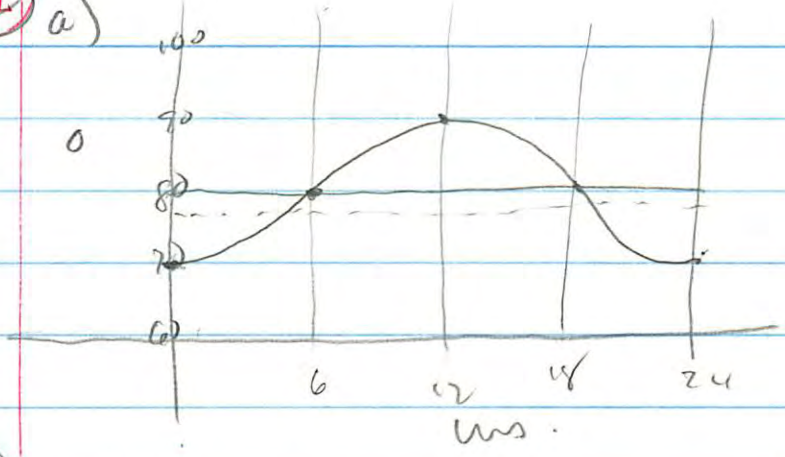
$=4.114$

$f(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right) \quad 0 \le t \le 24$

$\frac{2\pi}{\frac{\pi}{12}} = 24 = 1$ Period ⑤

② a)

0



axis of Symety $y = 80$

Amp 10 and Negative.

③ b) $\frac{1}{14-6}\int_6^{14} 80 - 10\cos\left(\frac{\pi t}{12}\right) dt \approx 87^\circ$ ① (70.1° if in degrees instead of Radians)

uses average fctn. ① proper limits ①

⑤ c) $f(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right) \overset{\text{Set}}{=} 78$ ①

$10\cos\left(\frac{\pi t}{12}\right) = 2$

$\cos\frac{\pi t}{12} = .2$

$\rightarrow \cos^{-1} .2 = \frac{\pi t}{12}$

$\frac{12}{\pi}\cos^{-1} .2 = t$

$t = 5.231 \leftarrow$ ①

Explanation method ②

because of Symetry $24 - 5.231 = 18.769 \leftarrow$ ①

is upper bound on time,

The Temperature is 78 or higher when $5.231 \le t \le 18.769$

①

⑤ D)

Explanation ③

$$\underbrace{.05}_{\text{cost per hr per degree}} \cdot \underbrace{(18.769 - 5.231)}_{\text{hrs at or above }78^\circ} \cdot \underbrace{\underbrace{\frac{1}{18.769-5.231}}_{\text{Length of time}} \underbrace{\int_{5.231}^{18.769} 80 - \cos\left(\frac{\pi t}{12}\right) - 78\,dt}_{\text{Total of Degrees at or above }78^\circ}}_{\text{Average degrees}}$$

① $.05\int_{5.231}^{18.769} 2 - 10\cos\left(\frac{\pi t}{12}\right) dt \approx 5.10$ ①

① The total cost to cool the home for the 24 hr period is about \$ 5.10

1998
⑥

$$2y^3 + 6\overset{u}{x^2}\overset{v}{y} - 12x^2 + 6y = 1$$

③ a) $6y^2\frac{dy}{dx} + 12xy + 6x^2\frac{dy}{dx} - 24x + 6\frac{dy}{dx} = 0$

$(6y^2 + 6x^2 + 6)\frac{dy}{dx} = 24x - 12xy$

$\frac{dy}{dx} = \frac{\cancel{12}^{2}(2x - xy)}{6(y^2 + x^2 + 1)} = \frac{4x - 2xy}{y^2 + x^2 + 1}$ ③

⑦ B) horizontal Tangent when $\frac{dy}{dx} = 0$ so set numerator of ①

$\frac{dy}{dx}$ [illegible]. $4x - 2xy \overset{set}{=} 0$

$2x(2 - y) = 0$

if x = 0 $\qquad x = 0$ or ① $y = 2$ $\qquad$ if y = 2

$16 + 12x^2 - 12x^2 + 12 = 1$

$28 = 1$

Reject

① $y \neq 2$

$2y^3 + 0 - 0 + 6y = 1$

$2y^3 + 6y - 1 = 0$ ③

From Solver $y \approx .165$ ③

$6y^2 + 6 = 0$

$6(y^2 + 1) > 0 \;\forall y \therefore y = .165$ is only zero. of the equation always increasing.

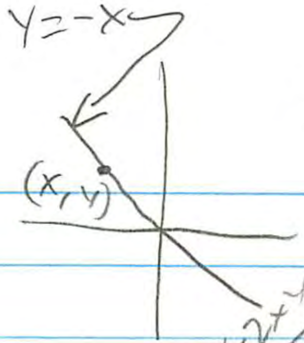
horizontal tangent is $y = .165$ ①

c) on Next page

1998
6
(continued)

⑤

(c) $2y^3+6x^2y-12x^2+6y=1$



Poor method

we know $x=-y$ ① and $\frac{dy}{dx}=-1$

$-2x^3-6x^3-12x^2-6x=1$ ①

$-8x^3-12x^2-6x=1$ ①

take der

method #1

$-24x^2-24x-6\overset{\text{set}}{=}0$

$-6(4x^2+4x+1)=0$

$-6(2x+1)^2=0$

$x=-\frac{1}{2}$ is $y=\frac{1}{2}$ | $(x=-y)$

$\frac{dy}{dx}=\frac{4x-2xy}{x^2+y^2+1}$

$\frac{4x+2x^2}{x^2+x^2+1}$

$\frac{2x(x+2)}{2x^2+1}$

Not justified well enough

So we know $(-\frac{1}{2},\frac{1}{2})$ is on curve but does the tangent line at that point: have slope -1?

yes. because we used $y=-x$.

method #2

$\frac{dy}{dx}=\frac{4x-2xy}{x^2+y^2+1}$ $\Big|_{y=-x}$ $=\frac{4x+2x^2}{2x^2+1}\overset{\text{set}}{=}-1$

Explain/method.

④ point P is on $y=-x$! Given.

$2x^2+4x=-2x^2-1$

$4x^2+4x+1=0$

$(2x+1)^2=0$

$x=-\frac{1}{2}$

when $x=-\frac{1}{2}$, $y=\frac{1}{2}$ since $y=-x$ and $(-\frac{1}{2},\frac{1}{2})$ and the tangent line does have slope -1.

①

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1997 REPORT TO AP® TEACHERS
CALCULUS AB
VACAVILLE HIGH SCHOOL

The following charts indicate the number of AP candidates from your school/district in each fifth of the total candidate group for each category described.

MULTIPLE-CHOICE

		LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN
DIFFERENTIAL CALCULUS	(23 Q.)	1	3	2	2	13	23	9.7	14.2
INTEGRAL CALCULUS	(17 Q.)	1	3	3	4	10	17	5.3	8.3
PART A- NO CALCULATOR	(25 Q.)	1	2	3	5	10	25	9.7	14.2
PART B- WITH CALCULATOR	(15 Q.)	1	2	4	5	9	15	5.2	8.4
WEIGHTED M-C SCORE		1	2	3	5	10	54	20.1	30.4
PERCENT IN EACH FIFTH		5%	10%	14%	24%	48%			
		LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN

AP GRADES	GRADE DISTRIBUTIONS	
	GLOBAL	GROUP
5	11.9%	47.6%
4	20.4%	23.8%
3	27.3%	4.8%
2	19.9%	19.0%
1	20.5%	4.8%
TOTAL CANDIDATES	111204	21

FREE-RESPONSE

	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN
QUESTION 1	2	2	3	5	9	9	4.5	6.0
QUESTION 2	3	2	5	6	5	9	5.5	6.5
QUESTION 3	6	1	4	4	6	9	6.7	6.9
QUESTION 4	2	2	3	6	8	9	4.5	5.8
QUESTION 5	2	2	2	4	11	9	2.0	4.6
QUESTION 6	##	##	##	##	##	9	1.2	3.2
WEIGHTED F-R SCORE	3	2		4	12	54	24.5	32.9
PERCENT IN EACH FIFTH	14%	10%		19%	57%			
	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN

DISTRIBUTION NOT PRINTED BECAUSE MORE THAN HALF OF THE TOTAL AP GROUP EARNED THE SAME SCORE

AI053595
VACAVILLE HIGH SCHOOL
AP COORD/MS JEAN BROOKS
100 MONTE VISTA AVENUE
VACAVILLE CA 95688

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Subject: Re: 1997 Part II Q6A
Date: 05/04 9:33 AM
Received: 05/04 3:05 PM
From: 112358etc, 112358etc@email.msn.com
Reply-To: Post AP Group, ap-calc@ets.org
To: Multiple recipients of list, ap-calc@ets.org

Jim:

You are right, and there is nothing lame in your explanation. The key is that differentiability of the solution (implied by the differential equation) implies that the solution is continuous. Thus it does not keep jumping back and forth across the input axis. Thus it is always positive or always negative over the interval of interest. Hence we can bring the minus sign (if any) across the equation and combine it with the constant e^A to obtain a new constant C.

>My question is, "How were we able to delete the absolute value bars?".
>My answer is that if v+16 is negative (as it is in this problem) then the
>C on the other side of the equation will take care of the negative and if
>v+16 is positive, then the C doesn't have anything "extra" to take of. I
>think my answer sounds pretty lame. I think it is correct --- but the
>lameness may be in the presentation.

Jeff Stuart
Department of Mathematics
University of Southern Mississippi
Hattiesburg, MS 39406 USA
jeff.stuart_spammers_are_bunions_on the soul_of society@usm.edu
(edit appropriately)

Subject: Re: 1997 part IB
Date: 05/04 8:37 AM
Received: 05/04 3:05 PM
From: LnMcmullin@aol.com
Reply-To: Post AP Group, ap-calc@ets.org
To: Multiple recipients of list, ap-calc@ets.org

In a message dated 4/30/99 10:49:24 Eastern Daylight Time, dkuhlmann@andover.edu writes:

<<
Jeff: Why do you say "this should be done on the calculator. Seems to me that students even in the calculator age should know the antiderfvative of ln(x) or at least be able to derive it using parts. I know that on the AP exam a numerical answer is often all that is asked for, but what happens if on the Free Response, say, the question says your answer must include an antiderivative?

Doug
>>

Doug,

If the question says the answer MUST contain an antiderivative then of course it MUST contain an antiderivative. That's not likely to happen this year since symbolic antiderivative may be easily obtained on TI89 and various HP calculators. Questions involving definite integrals on Part II should show the definite integral and the numerical answer -- it is assumed that the integration will be done on a calculator since that is one of the four things you may do on a calculator. In fact it would not be smart to do it any other way (think about it: 3, 4, 5 or more lines of computation for 1 point). And, WADR if your question is a comment about calculator use, that's the way it should be done in the "Real World" as well.

The testing of finding antiderivative without a calculator is done on the non-calculator part of Part I. It is there; it will be tested. Questions are not put on the exam willy-nilly: the exam committee is very aware of when a calculator can and cannot be used and arrange the exam accordingly. Next year (2000) there will also be 3 calculator prohibited questions on the Free Response section where antidifferentiation by hand may be required.

Lin McMullin
Ballston Spa, NY

Subject: Re: 1997 Part II Q6A
Date: 05/04 7:58 AM
Received: 05/04 3:05 PM
From: Richard Sisley, keckcalc@earthlink.net
Reply-To: Post AP Group, ap-calc@ets.org
To: Multiple recipients of list, ap-calc@ets.org

Jim wrote:

>
> In the book with the solutions (and questions) to the 1997 exam (on page
> 145) shows the solution to a separable differential equation.
>
> On the third line of the solution they have ln abs(v + 16) = - 2t + A.
> The next line has abs (v+16) = e^(-2t+A) = e^A e^(-2t)
> so far so good. e has been raised to the exponent powers of the equal
> items and e^A will be recognized as a constant C on the next line.
> The fifth line has v + 16 = Ce^(-2t).
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> My question is, "How were we able to delete the absolute value bars?".
> My answer is that if v+16 is negative (as it is in this problem) then the
> C on the other side of the equation will take care of the negative and if
> v+16 is positive, then the C doesn't have anything "extra" to take of. I
> think my answer sounds pretty lame. I think it is correct --- but the
> lameness may be in the presentation.
>
> Your thoughts would be appreciated.
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> Jim Mumaugh

Jim, I agree that this is not ideal (lame, if you will). It should seem arbitrary to many good students that a second "arbitrary" constant has been introduced. I found in discussing this solution with my students that many thought first that "C" was being used as a equivalent to e^A. Also, "C" is only semi-arbitrary, since it makes no sense in the context of the problem to replace "C" with a non-negative real number. I just do not like this sudden restriction on an "arbitrary" constant.

When we discussed this problem in class, we thought aboutthe physical reality of the situation and its representation given the frame of reference implied by making the initial velocity negative. You should know that there was an earlier series of postings on this problem in which I gave full details and several others commented.

I certainly encouraged my students to face the absolute value issue head on. In fact I gave them a follow up quiz with a question involving Newton's Law of Temperature change in which the issue had to be faced to get a solution which fit the situation in which a temperature was increasing instead of decreasing.

Sincerely,

Richard Sisley
keckcalc@earthlink.net

Subject: Volume
Date: 05/04 9:12 AM
Received: 05/04 3:05 PM
From: LnMcmullin@aol.com
Reply-To: Post AP Group, ap-calc@ets.org
To: Multiple recipients of list, ap-calc@ets.org

-

In a message dated 4/30/99 11:51:48 Eastern Daylight Time, keckcalc@earthlink.net writes:

<< A volume accumulation function can be defined as the function whose graph is made up of points described as (the position of the sweeping surface, the volume of the region through which the surface has swept in reaching that position). It is remarkably easy to make a convincing case for the validity of the general principle that the rate of accumulation of volume at any point in the sweep is the area of the sweeping surface at that point. >>

Richard,

An interesting idea. I expect you also show the unit analysis:
d(Volume)/d(one dimension) = rate of change of volume. Units:
(distance)^3/(Distance)= (distance)^2 = Area.
Integral (Rate of change) = (Amount of change)
Integral (Area) = Volume

Lin McMullin
Ballston Spa, NY

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Reply-To: Post AP Group, ap-calc@ets.org
To: Multiple recipients of list, ap-calc@ets.org

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Sincerely,

Richard Sisley
keckcalc@earthlink.net

Subject: Re: 1997 Part II Q1B
Date: 05/04 9:17 AM
Received: 05/04 3:05 PM
From: 112358etc, 112358etc@email.msn.com
Reply-To: Post AP Group, ap-calc@ets.org
To: Multiple recipients of list, ap-calc@ets.org

Jim:

The short answer is that VELOCITY is signed. If you travel at +15 miles per hour for an hour and then at -15 miles per hour for an hour, your average velocity is ZERO MILES PER HOUR (Don't forget units on the exam!!) Your average speed for the two hour period is 15 miles per hour.

-----Original Message-----
From: Jim <jamesm98@slip.net>
To: Multiple recipients of list <ap-calc@ets.org>
Date: Monday, May 03, 1999 4:40 PM
Subject: 1997 Part II Q1B

The question references "average velocity" and the intended solution involves the "average value of a function" integral because the given function was velocity.

When we were discussing the 97 test a question came up concerning "average velocity". The question had to do with the definition of "average velocity". They wanted to use "total distance divided by total time" to give "average velocity". In this problem, the total distance was found in part C and was 17. The net distance as computed using the integral is 15.

Why isn't the "total distance divided by total time" approach the proper method? This method requires student to recognize that the particle moved first left then right -- a fact which is ignored using the integral version.

Thanks for your thoughts.

Jim Mumaugh

Subject: HA: 1997 Part II Q1B
Date: 05/04 9:23 AM
Received: 05/04 3:05 PM
From: Basil Silantyev, basyka@www.medcenter.msk.ru
Reply-To: Post AP Group, ap-calc@ets.org
To: Multiple recipients of list, ap-calc@ets.org

The thing is the definition of the so called "total distance". By total distance we mean the "length of the whole path travelled". That is not just the integral of the velocity over the whole time interval, but the integral of the absolute value of the velocity. In this definition all changes in direction are already taken into account. And the integral of the velocity which is equal to (x_final-x_initial)/(time_interval) is the length of the "so called transition vector".

Best regards
Basil Silantyev

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..

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Thanks for your thoughts.

Jim Mumaugh

Subject: Re: 1997 Part II Q1B
Date: 05/04 9:28 AM
Received: 05/04 3:05 PM
From: Richard Sisley, keckcalc@earthlink.net
Reply-To: Post AP Group, ap-calc@ets.org
To: Multiple recipients of list, ap-calc@ets.org

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>
> Thanks for your thoughts.
>
> Jim Mumaugh

I think it is helpful to make a distinction between the net change of position over an interval of time and the distance traveled over that interval of time. If there is any change of direction during that time, the two measures will not be the same. An telling case is when the position is say the sine function and the time interval is 0 to 2 pi. Here the net change of position is 0 and the average rate of change of position, also called the average velocity, is 0. However, the distance traveled is 4. I think it is correct to refer to 4/(2Pi) as the average speed as opposed to the average velocity. This fits well with the idea that speed numbers are the absolute value of velocity numbers.

There is also a related set of ideas related to integrals and areas. The change of position is equal the the integral of the velocity function over the time interal. Dimensional analysis supports this--just think about forming Riemann Sums to approximate the change of position. On the other hand, the total distance traveled is equal to the area of the region bounded by vertical segments at the beginning of and end of the time interval, the velocity function and the interval of the time axis. This is true whether the velocity function graph of above or below the time axis. In fact, this area is equal to the area bounded above by the absolute value of the velocity function--in other words it is the area of the region bounded above by the speed function and below by the relevant interval of the time axis.

This is a nice bunch of connected ideas.

Hope this is of some help.

1. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position $x(t)$ is 5 for $t = 2$.

   (a) Write a polynomial expression for the position of the particle at any time $t \geq 0$.

   (b) For what values of t, $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval $[0, 3]$?

   (c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

(a) $x(t) = \int v(t)\,dt = \int (3t^2 - 2t - 1)\,dt$

$= t^3 - t^2 - t + C$

$x(2) = 8 - 4 - 2 + C = 5$; $\quad C = 3$

$x(t) = t^3 - t^2 - t + 3$

3
- 2: $x(t) = t^3 - t^2 - t + C$
  - <−1> error(s) in $t^3 - t^2 - t$
  - <−1> no constant of integration
- 1: evaluates constant of integration

(b) avg. vel. $= \dfrac{x(3) - x(0)}{3 - 0}$

$= \dfrac{18 - 3}{3} = 5$

$3t^2 - 2t - 1 = 5$

$t = \dfrac{1 + \sqrt{19}}{3}$ or 1.786

3
- 1: average velocity = 5
- 1: sets $v(t)$ equal to student's average velocity
- 1: answer
  0/1 if not solving
  $3t^2 - 2t - 1 =$ an avg. velocity

(c) distance $= \int_0^3 |v(t)|\,dt$

$= \int_0^3 |3t^2 - 2t - 1|\,dt = 17$

or

$v(t) = 3t^2 - 2t - 1 = 0$

$t = -\dfrac{1}{3}$, $t = 1$

$x(0) = 3$

$x(1) = 1 - 1 - 1 + 3 = 2$

$x(3) = 27 - 9 - 3 + 3 = 18$

distance $= (3 - 2) + (18 - 2) = 17$

3
- 1: limits of 0 and 3 on an integral of $v(t)$ or $|v(t)|$
  or
  substitutes $t = 0$ and $t = 3$ in $x(t)$
- 1: handles change in direction at student's turning point
- 1: answer

AB-2 1997

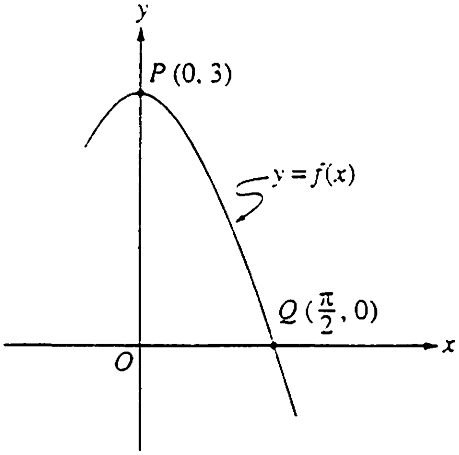
2. Let f be the function given by $f(x) = 3\cos x$. As shown above, the graph of f crosses the y-axis at point P and the x-axis at point Q.

   (a) Write an equation for the line passing through points P and Q.

   (b) Write an equation for the line tangent to the graph of f at point Q. Show the analysis that leads to your equation.

   (c) Find the x-coordinate of the point on the graph of f, between points P and Q, at which the line tangent to the graph of f is parallel to line PQ.

   (d) Let R be the region in the first quadrant bounded by the graph of f and line segment PQ. Write an integral expression for the volume of the solid generated by revolving the region R about the x-axis. Do not evaluate.



(a) slope $= \dfrac{3-0}{0-\pi/2} = -\dfrac{6}{\pi}$ $y - 3 = -\dfrac{6}{\pi}(x-0)$	2 { 1: slope 1: equation
(b) $f'(x) = -3\sin x$ $f'(\pi/2) = -3\sin(\pi/2) = -3$ $y - 0 = -3(x - \pi/2)$	2 { 1: $f'(\pi/2) = -3$ 1: equation using $f'(\pi/2)$ and $f(\pi/2)$ 1/2 if equation only
(c) $f'(x) = -3\sin x = -\dfrac{6}{\pi}$ $\sin x = \dfrac{2}{\pi}$ $x = 0.690$	2 { 1: equates derivative to slope 1: solution in $[0, \pi/2]$ 1/2 if solution only
(d) $V = \pi\displaystyle\int_0^{\pi/2}\left[(3\cos x)^2 - \left(-\frac{6}{\pi}x + 3\right)^2\right]dx$	3 { 2: integrand 0/2 if not difference of 2 squares 1/2 if incorrect but of form $(a\cos x)^2 - (bx+c)^2$; $a, b \neq 0$ 1/2 if reversal 1: constant and limits

QUESTION AB 3 and STANDARDS

AB-3

1997

3. Let f be the function given by $f(x) = \sqrt{x-3}$.

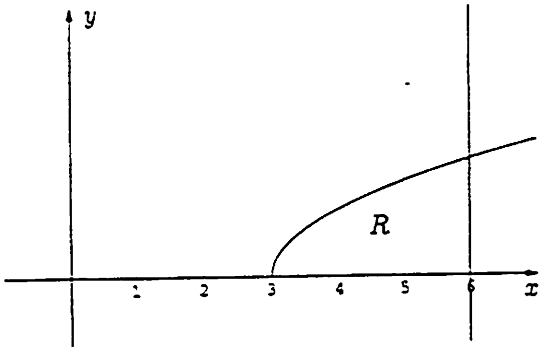
   (a) On the axes provided below, sketch the graph of f and shade the region R enclosed by the graph of f, the x-axis, and the vertical line $x = 6$.

   (b) Find the area of the region R described in part (a).

   (c) Rather than using the line $x = 6$ as in part (a), consider the line $x = w$, where w can be any number greater than 3. Let $A(w)$ be the area of the region enclosed by the graph of f, the x-axis, and the vertical line $x = w$. Write an integral expression for $A(w)$.

   (d) Let $A(w)$ be as described in part (c). Find the rate of change of A with respect to w when $w = 6$.

(a)



2
- 1: graph of f, (domain is $x \geq 3$, goes through $(3, 0)$, is increasing, positive, and concave down)
- 1: correct region relative to graph of f

(b) $$\text{area} = \int_3^6 \sqrt{x-3}\,dx = \frac{2}{3}(x-3)^{3/2}\Big]_3^6$$

$$= 2\sqrt{3} = 3.464$$

3
- 1: limits
- 1: integrand
- 1: answer
  0/1 if second point is not earned

(c) $$A(w) = \int_3^w \sqrt{x-3}\,dx$$

2
- 1: limits
- 1: integrand

(d) $$\frac{dA}{dw} = \sqrt{w-3}$$

$$\left.\frac{dA}{dw}\right|_{w=6} = \sqrt{3} = 1.732$$

2
- 1: $\frac{dA}{dw}$
- 1: evaluation at 6

0/2 if $\frac{dA}{dw}$ is constant

AB-5. BC-5 1997

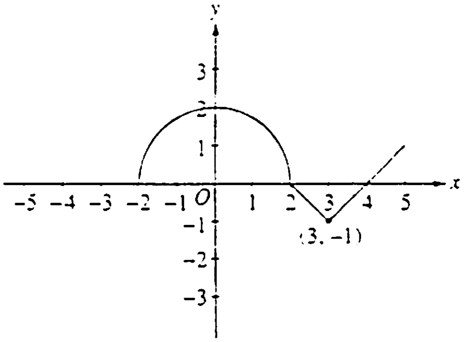
5. The graph of a function f consists of a semicircle and two line segments as shown above. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(3)$.

(b) Find all values of x on the open interval $(-2, 5)$ at which g has a relative maximum. Justify your answer.

(c) Write an equation for the line tangent to the graph of g at $x = 3$.

(d) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-2, 5)$. Justify your answer.



(a) $g(3) = \int_0^3 f(t)\,dt$

$$= \frac{1}{4}\cdot\pi\cdot 2^2 - \frac{1}{2} = \pi - \frac{1}{2}$$

$$2\begin{cases}\text{2: answer}\\ \quad\langle -1\rangle \text{ each incorrect area}\\ \quad\langle -1\rangle \text{ error in summing}\end{cases}$$

(b) $g(x)$ has relative maximum at $x = 2$

because $g'(x) = f(x)$ changes from positive to negative at $x = 2$

$$3\begin{cases}\text{1: relative maximum at } x = 2 \text{ only}\\ \text{1: } g'(x) = f(x) \text{ or interprets}\\ \quad g(x) \text{ as area accumulator}\\ \text{1: justification}\\ \quad\text{(ignore discussion at } x = 5)\end{cases}$$

(c) $g(3) = \pi - \dfrac{1}{2}$

$g'(3) = f(3) = -1$

$$y - \left(\pi - \frac{1}{2}\right) = -1(x - 3)$$

$$2\begin{cases}\text{1: } g'(3) = -1\\ \text{1: equation using } g(3) \text{ and } g'(3)\end{cases}$$

(d) graph of g has points of inflection with x-coordinates $x = 0$ and $x = 3$

because $g''(x) = f'(x)$ changes from positive to negative at $x = 0$ and from negative to positive at $x = 3$

or

because $g'(x) = f(x)$ changes from increasing to decreasing at $x = 0$ and from decreasing to increasing at $x = 3$

$$2\begin{cases}\text{1: points of inflection with}\\ \quad x\text{-coordinates 0 and 3 only}\\ \text{1: justification}\\ \quad\text{(ignore discussion at } x = 2)\end{cases}$$

1/2 if $x = 0, 3$ selected as candidates and $x = 3$ discarded because $g''(3)$ does not exist

1/2 if $x = 0, 2, 3$ selected as candidates and $x = 2$ and $x = 3$ discarded because $g''(2)$ and $g''(3)$ do not exist

QUESTION AB 6, BC 6 and STANDARDS

AB-6, BC-6 1997

6. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

   (a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

   (b) Terminal velocity is defined as $\lim_{t\to\infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

   (c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

(a) $\frac{dv}{dt} = -2v - 32 = -2(v + 16)$

$$\frac{dv}{v+16} = -2\,dt$$

$$\ln|v + 16| = -2t + A$$

$$|v + 16| = e^{-2t+A} = e^{A}e^{-2t}$$

$$v + 16 = Ce^{-2t}$$

$$-50 + 16 = Ce^{0}\,; \quad C = -34$$

$$v = -34e^{-2t} - 16$$

6
- 1: separates variables
- 1: antiderivative of dv side
  - 0/1 if not $\int \frac{dv}{av+b}$, $a \neq 0$
- 1: antiderivative of dt side
- 1: constant of integration
- 1: uses initial condition $v(0) = -50$
- 1: solves for $v(t)$
  - 0/1 if not solving $\frac{dv}{av+b} = k\,dt$ where a, b, k nonzero
  - 0/1 if no constant of integration

0/6 if variables not separated

(b) $\lim_{t\to\infty} v(t) = \lim_{t\to\infty}\left(-34e^{-2t} - 16\right) = -16$

1: limit value
must be exponential $v(t)$ with finite limit

(c) $v(t) = -34e^{-2t} - 16 = -20$

$$e^{-2t} = \frac{2}{17}\,; \quad t = -\frac{1}{2}\ln\left(\frac{2}{17}\right) = 1.070$$

2
- 1: sets $v(t) = -20$
- 1: solution
  - must be exponential $v(t)$

ANALYSIS OF INDIVIDUAL QUESTIONS

Why isn't average velocity Total Dist divided by Total time? In this problem the total Dist is 17 and the time is 3 so 17/3 would be average velocity a rate. Why does $\int |v(t)| dt$ give proper distance?

AB-1: Students did well on this question.

(a) The student who forgot the constant lost 2 points.

(b) The second of the three (3) points was a *soft* point since v(t) was provided in the stem of the problem. Saying that v(t) = "whatever" the student believes to be the average value earned the point. The first and third points were tougher to earn, however, because the student had to correctly find the average velocity first and then use the average velocity, correct or incorrect, to get the quadratic equation to solve for the third point.

(c) The student using the calculator to evaluate the definite integral for total distance had the advantage because a correct setup and final answer earned all three (3) points. Those doing all or part by hand additionally had to recognize the change of direction in the interval from 0 to 3 and handle it properly.

Commentary: The turning point of t = 2 kept a number of students from missing this problem. These students calculated the position on the particle for integral values of t and would have missed the turning point if it had been at a non-integral value.

AB-2:

(a) No work had to be shown for the slope of -6/π. The answer presented in the form (y-3)/(x-0) = -π/6 was not acceptable.

(b) No points were awarded if there was no indication that the derivative was used to determine the slope.

(c) A claim of "no answer" did not earn the solution point.

(d) Students were able to "import" an incorrect linear equation from (a) and still earn the two (2) points for the correct integrand. Some students used the shell method to solve this problem.

Commentary: Students who rounded off to three(3) decimal places for intermediate values did not get the final answers correct to three (3) decimal places. Student should carry as many decimals as possible until the final answer is obtained and round off at this point. Some readers felt that a straightforward application of the mean value theorem did not make for a "very interesting" problem.

AB-3: This question was one of the highest scored questions in the history of AB Calculus. Students had to do very little to earn all 9 points.

(a) The calculator could quickly show the graph, and those using it who lost points did so mostly because they made errors with parentheses.

(b) The correct setup and final answer(from the calculator) earned all 3 points. Those who just gave only the answer got only one point because the directions clearly state the importance of showing the setup of work leading to the answer. THIS WILL CONTINUE TO BE A POINT OF EMPHASIS.

(c) Both points could be earned by copying the integrand from (b) and replacing the 6 with a w.

(d) Notational errors could be costly, particularly those which indicated that the student thought that he was doing a related rate problem. A lot of *dr*'s and *dt*'s suddenly appeared in student solutions.
Commentary: Some readers felt that some students presented very good solutions to this problem but were penalized too much for round off errors. The best solutions for this problem were frequently those which were the shortest.

AB-4:

(a) If the solution of f'(x) = 0 yielded only one solution, the student was not eligible for the 3rd and 4th points for this part.
(b) Any supported correct answer for this part earned both points. The following answers were accepted as correct.

"Between 0 and 32." "0 to 32." "0 through 32."

The following answers were not accepted as correct.

"$0 > p < 32$" "$0 < p > 32$" "$0 > p > 32$"
(c) The student received no points for this part if no integration appeared in the solution. If the solution did not include the constant 1/3 in front of the integral sign and a p value of 61/12 or 5.0833... was obtained, the student earned two (2) of the three (3) points for this part of the problem. A correct unsupported answer earned only one (1) of the three (3) points.

AB-5, BC-5: (The Hump Check Problem). This was a good question which read slowly at first but picked up speed later on. Many students did not recognize that the integral from -2 to 0 is negative. Many students tried to draw the graph of g(x), but drew it incorrectly.
a) Many students added two positive numbers together, e.g., $\pi + 1/2$, not seeing the difference between the integral and the total area.
(b) The key was the recognition that g'(x) = f(x) or that g(x) accumulates the area under f(x). There was a lot of discussion to read through for the justification. *Unlabeled graphs and unlabeled sign tests* resulted in lost justification points.
(c) There were two ways a student could get the slope of -1: the correct way g'(3) = f(3) = -1 or by looking at the slope of the line segment from 2 to 3 on the graph of f(x). Readers had to figure out which one was being used. Students who used the point-slope form of a line had an easier time giving the correct equation.
(d) It was not sufficient to say that g(x) has inflection points at 0 and 3 because g"(x) = f'(x) = 0 at those points. A two-sided argument was required to earn the justification point.

Commentary: Students often mixed up the y-value and the slope value of -1, and the x-value and the maximum value of 2 in this problem.

AB-6, BC-6: The Skydiver Problem

(a) If the student did not separate the variables, he was not eligible for any of the six (6) points for this part. If the separation of variables was incorrect but of the form f(t) *dt* = g(v) *dv*, the student was eligible for some credit in this part. The constant of integration, A, has to appear when it does in the standard. The absolute value symbols must appear if the expression following the *ln* negative for v = -50. The solution was not eligible for the 4th and 6th points if there was not a constant of integration. A student who determined a correct expression for v using integrating factors earned five (5) of the six (6) points.

(b) A correct unsupported answer earned the point for this part. A correct answer which used the student's incorrect expression from (a) earned the answer point.

(c) The student had to show the equation being solved if he used a graphing calculator to answer this part. If the student's equation for v from (a) was an exponential function and was solved correctly, the student earned the answer point. The student was also eligible for the answer point if using v = +20 led to a positive answer for **t**.

Commentary: The +/- associated with up and down was not clear in this problem. Students who solved this problem using integrating factors and not the technique of separation of variables were penalized one (1) point for not solving the problem by the designated method. This disturbed some readers.

BC-1.

(a) A student was given full credit for the answer for this part if found with a calculator.

(b) The graph had to closely resemble an ellipse, and no diamonds, hexagons, or concave-in curves were accepted. The intercepts of the graph were carefully checked. One arrow to indicate direction was sufficient.

(c) A bald answer of 3 was enough. It could be found by tracing, analyzing the period, or solving for **t**. If a student gave **t** values, all **t** values had to be correct.

(d) No comment.

(e) A student could lose a point for incorrectly simplifying the integrand but could earn the answer point if his answer followed from his integral. Some students used calculators with built-in arc length functions to arrive at the correct numerical answer. If the value obtained in this manner did not match the student's definite integral, no credit was earned for the answer point.

BC-2:

(a) This was a straightforward problem.

(b) Some students were careless about putting the exponent 2 on the third term of the expression for $P'3(x) = 3 + 10(x - 4) - 6(x - 4)^2$. There was no deduction for using ≈ or =.

(c) The student got the first point if he indicated in any way that he was performing integration unless the wrong function was being used. Some students multiplied out each of the terms which was not a good use of time and was a source of errors in some cases, costing the student the answer point.

(d) A student earned the second point, but not the first, if he answered "yes" and then stated that the values may be approximate and not exact. If the student answered "no" and then stated that the values were "only approximate" or "not exact," she also earned the second point. The student did not earn the second point if she answered "no" and then stated that 3 was "too far" from 4 or the polynomial should be centered at 3.

Commentary: Some readers felt that there was a lack of clarity about the approximation in this problem.

BC-3:

(a) The integrand had to involve the difference of two functions and the answer value had to follow from the written integral. If a student reversed the terms in the integrand, he was required to use absolute value bars around the integrand or offer an explanation for ignoring the negative sign obtained in this case. Integral limits had to show three (3) decimal places as did the answer.

(b) If a student could not find the x-coordinates of the points of intersection or found them incorrectly, consistent use of the integral limits used in (a) helped earn points in this part. An indefinite integral lost one (1) point. Use of a constant a instead of 0.91586 in this part was permissible. The integrand did not have to be simplified, but no credit was earned for a symbolic definition.

(c) The comment for (b) applies to this part as well.

BC-4:

(a) This was an easy point for the student to earn.

(b) The student did not earn the second point if the slope was a constant. The student did not earn the third point if he used the form $y = mx + b$ and did not determine the value of b. Also, the student did not earn the third point if he used the form $y - y_1 = m(x - x_1)$ and did not have values substituted into the slope.

(c) The bounds of integration had to agree with the differential variable or the variable in the integrand. If the student doubled the area, the error point came out of the integrand point. In order to earn the fourth point, the student had to state that as k approaches ∞, the expression $k^{-1.5}$ approaches 0, and the area approaches 0.

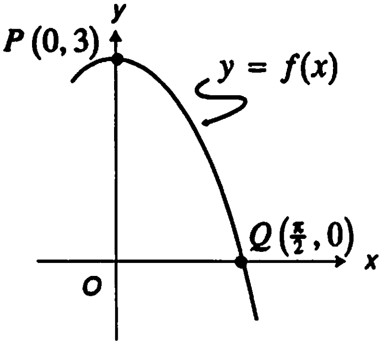
1997 Advanced Placement Calculus AB

1. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position $x(t)$ is 5 for $t = 2$.

   a) Write a polynomial expression for the position of the particle at any time $t \geq 0$.

   b) For what values of t, $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval [0 , 3] ?

   c) Find the total distance traveled by the particle from time $t = 0$ until $t = 3$.



2. Let f be the function given by $f(x) = 3\cos x$. As shown, the graph of f crosses the y-axis at point P and the x-axis at point Q.

   a) Write an equation for the line passing through points P and Q.

   b) Write an equation for the line tangent to the graph of f at point Q. Show the analysis that leads to your equation.

   c) Find the x-coordinate of the point on the graph of f, between points P and Q, at which the line tangent to the graph of is parallel to line PQ.

   d) Let R be the region in the first quadrant bounded by the graph of f and line segment PQ. Write an integral expression for the volume of the solid generated by revolving the region R about the x–axis. Do not evaluate.

3) Let f be the function given by $f(x) = \sqrt{x - 3}$.

   a) Sketch the graph of and shade the region R enclosed by the graph of f, the x-axis, and the vertical line $x = 6$

   b) Find the area of the region described in part (a).

   c) Rather than using the line $x = 6$ as in part (a), consider the line $x = w$, where w can be any number greater than 3. Let $A(w)$ be the area of the region enclosed by the graph of f, the x-axis, and the vertical line $x = w$. Write an integral expression for $A(w)$.

   d) Let $A(w)$ be as described in part (c). Find the rate of change of A with respect to w when $w = 6$.

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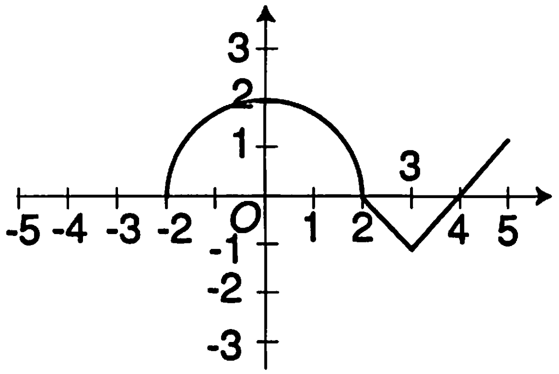
4) Let f be the function given by $f(x) = x^3 - 6x^2 + p$, where p is an arbitrary constant.

a) Write an expression for $f'(x)$ and use it to find the relative maximum and minimum values of f in terms of p. Show the analysis that leads to your conclusion.

b) For what values of the constant p does f have 3 distinct real roots?

c) Find the value of p such that the average value of f over the closed interval $[-1, 2]$ is 1.



5) The graph of a function f consists of a semicircle and two line segments as shown. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

a) Find $g(3)$.

b) Find all values of x on the open interval $(-2, 5)$ at which g has a relative maximum. Justify your answer.

c) Write an equation for the line tangent to the graph of g at $x = 3$

d) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-2, 5)$. Justify your answer.

6. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

b) Terminal velocity is defined as $\lim_{t \to \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

c) It is safe to land when her speed is 20 feet per second. At what time does she reach this speed?

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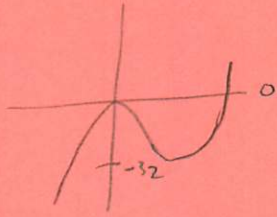
① c) 17

② c) $x = .690 = \sin^{-1}\left(\frac{2}{\pi}\right)$

③ d) $\frac{dA}{dt} = \sqrt{3} = 1.732$

④ B $0 < P < 32$ must explain



⑤ c) $y = -x + \frac{5}{2} + \pi$

OR

$y = -x + 5.642$

⑥ a) $v(t) = -34e^{-2t} - 16$

Jeannette Lynn Dickerson

FORM 3TBP

1997

The College Board
Advanced Placement Examination

CALCULUS AB
SECTION II

This green insert may be used for reference and/or scratchwork as you answer the free-response questions, but be sure to show all your work and your answers in the pink booklet. No credit will be given for work shown on this green insert.

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CALCULUS AB

SECTION II

Time — 1 hour and 30 minutes
Number of problems — 6
Percent of total grade — 50

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

General instructions for this section are printed on the back cover of the test booklet.

---

1. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position $x(t)$ is 5 for $t = 2$.

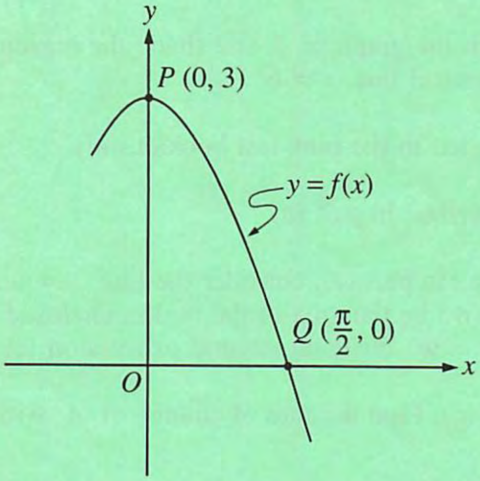
   (a) Write a polynomial expression for the position of the particle at any time $t \geq 0$.

   (b) For what values of t, $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval $[0, 3]$?

   (c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

$\int_0^3 \sqrt{1+}$

GO ON TO THE NEXT PAGE



2. Let f be the function given by $f(x) = 3\cos x$. As shown above, the graph of f crosses the y-axis at point P and the x-axis at point Q.

(a) Write an equation for the line passing through points P and Q.

(b) Write an equation for the line tangent to the graph of f at point Q. Show the analysis that leads to your equation.

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(d) Let R be the region in the first quadrant bounded by the graph of f and line segment PQ. Write an integral expression for the volume of the solid generated by revolving the region R about the x-axis. Do not evaluate.

---

slope = $\frac{-P}{Q}$

y-intercept = P

GO ON TO THE NEXT PAGE

3. Let f be the function given by $f(x) = \sqrt{x - 3}$.

(a) On the axes provided below, sketch the graph of f and shade the region R enclosed by the graph of f, the x-axis, and the vertical line $x = 6$.

Note: The axes for this graph are provided in the pink test booklet only.

(b) Find the area of the region R described in part (a).

(c) Rather than using the line $x = 6$ as in part (a), consider the line $x = w$, where w can be any number greater than 3. Let $A(w)$ be the area of the region enclosed by the graph of f, the x-axis, and the vertical line $x = w$. Write an integral expression for $A(w)$.

(d) Let $A(w)$ be as described in part (c). Find the rate of change of A with respect to w when $w = 6$.

$\frac{dv}{dt} = -2v - 32$

$v'(0) = -32$

$\frac{dv}{dt} + 2v = -32$

$\frac{2}{3}(x-3)^{3/2}$

$2\,\frac{3}{2}(x-3)^{1/2}$

$A(w) = \int \cos x = \sin x$

$\int \cos x =$

$\sin x$

$\int$

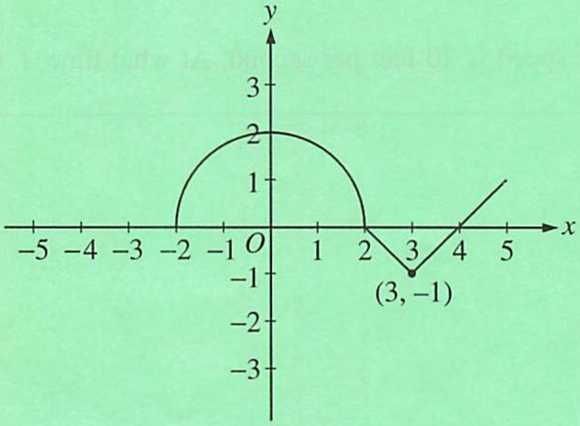
GO ON TO THE NEXT PAGE

4. Let f be the function given by $f(x) = x^3 - 6x^2 + p$, where p is an arbitrary constant.

(a) Write an expression for $f'(x)$ and use it to find the relative maximum and minimum values of f in terms of p. Show the analysis that leads to your conclusion.

(b) For what values of the constant p does f have 3 distinct real roots?

(c) Find the value of p such that the average value of f over the closed interval $[-1, 2]$ is 1.



5. The graph of a function f consists of a semicircle and two line segments as shown above. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(3)$.

(b) Find all values of x on the open interval $(-2, 5)$ at which g has a relative maximum. Justify your answer.

(c) Write an equation for the line tangent to the graph of g at $x = 3$.

(d) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-2, 5)$. Justify your answer.

$f'(x)=$

$0 < p < 32$

GO ON TO THE NEXT PAGE

6. Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

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(c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

$v(0)$

END OF EXAMINATION

$\frac{dv}{dt} = -2v - 32$

$a = -2\frac{dv}{dt}$

a'

1997

①

$v(t) = 3t^2 - 2t - 1 \qquad x(2) = 5 \qquad t \ge 0$ ✻

④ a) $x(t) = \int v(t)$ ①

$v = 3t^2 - 2t - 1$

$\int dv = \int 3t^2 - 2t - 1 \, dt$

$x = t^3 - t^2 - t + C$ ①

$x(2) = 8 - 4 - 2 + C \overset{set}{=} 5$ ①

$2 + C = 5$

$C = 3$

$x(t) = t^3 - t^2 - t + 3$ ①

⑥ b) $0 \le t \le 3$

$v \overset{?}{=} \text{ave vel}$

ave veloc $\frac{x(3) - x(0)}{3 - 0}$ ①

$x(3) = 27 - 9 - 3 + 3$

$= 18$ ①

$x(0) = 3$ ①

$\frac{18 - 3}{3 - 0} = 5 = \text{Ave Vel}$

$3t^2 - 2t - 1 \overset{set}{=} 5$ ①

$3t^2 - 2t - 6 = 0$

$t = \frac{1 + \sqrt{19}}{3} = 1.786$ ②

⑤ c) $v(t) = 3t^2 - 2t - 1$

② $= (3t + 1)(t - 1)$

$= t \in \{-\frac{1}{3}, 1\}$

v — 0 — 1 : − 0 +

③ particle starts at 3, goes Left until $t = 1$ ($x(1) = 1 - 1 - 1 + 3 = 2$) so goes from 3 to 2. One unit, then from $x = 2$ to $t = 3$ ($x = 18$) 16 units

total Distance traveled $16 + 1 = 17$ unit.

— OR —

→ fnint(Abs($3t^2 - 2t - 1$), x, 0, 3) = 17.

✻ Velocity is signed. Speed is Not, if You travel +15 mph for 1 hr then −15 mph for one hour Your average Velocity is Zero mph And Your average Speed for The 2 hrs is 15 mph

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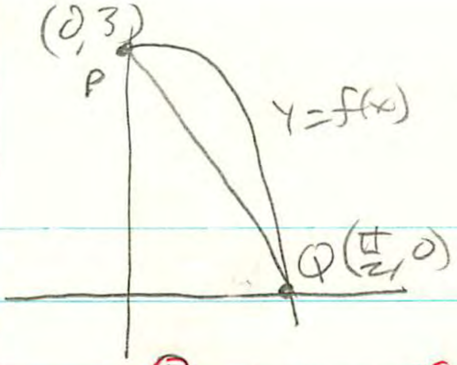
②



$y = 3\cos x$

④ a) $m = \overset{①}{\dfrac{3-0}{0-\frac{\pi}{2}}}$

$= \dfrac{3}{-\frac{\pi}{2}}$

$= -\dfrac{6}{\pi}$ ①

Y intercept is 3 ①

$\therefore\ y = -\dfrac{6}{\pi}x + 3$ ①

④ b) Tangent line on

$m = \dfrac{dy}{dx} = -3\sin x \Big|_{x=\frac{\pi}{2}} = -3$ ②

$\left(\dfrac{\pi}{2}\ 0\right)$ ①

$y = -3\left(x - \dfrac{\pi}{2}\right)$

$y = -3x + \dfrac{3\pi}{2}$ ①

Set slopes = for parallel

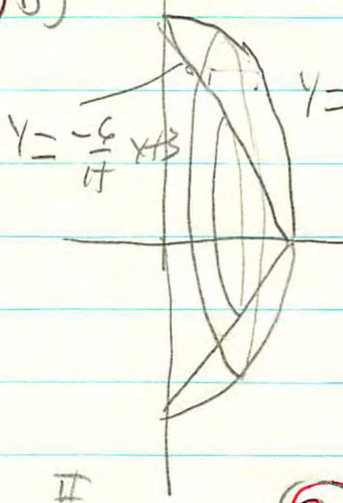
④ c) $-3\sin x \overset{\text{set}}{=} -\dfrac{6}{\pi}$ ②

$\sin x = \dfrac{2}{\pi}$

① Solve$(\sin x = \frac{2}{\pi}, x, .1)$

① $x = .690$

③ D) "washers" ①

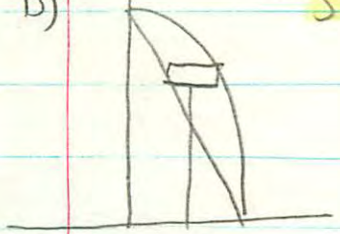


$\pi \displaystyle\int_a^b r_1^2 - r_2^2 \, dx$

$\pi \displaystyle\int_0^{\frac{\pi}{2}} 9\cos^2 x - \left(-\frac{6}{\pi}x + 3\right)^2 dx$ ②

— or —

D) "shells"



$y = 3\cos x$

$\dfrac{y}{3} = \cos x$

$\cos^{-1}\left(\dfrac{y}{3}\right) = x$

$2\pi \displaystyle\int_0^3 y\left(\cos^{-1}\left(\frac{y}{3}\right) - \left(-\frac{\pi}{6}y + \frac{\pi}{2}\right)\right) dy$

$y = -\dfrac{6}{\pi}x + 3$

$x = -\dfrac{\pi}{6}y + \dfrac{\pi}{2}$

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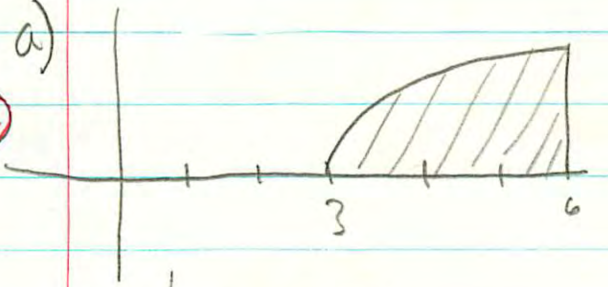
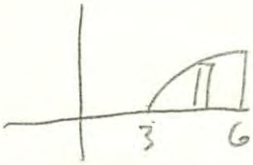
③

$f(x) = \sqrt{x-3}$

a) ②



⑤ b)

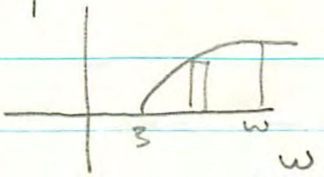


$\int_3^6 \sqrt{x-3}\,dx$ ②

$\frac{2}{3}(x-3)^{\frac{3}{2}}\Big|_3^6 =$ ② Proper integration

$\frac{2}{3}\left(3^{\frac{3}{2}}\right) = \frac{2\cdot 3^{\frac{3}{2}}}{3^{\frac{2}{2}}} = 2\sqrt{3}$ ①

correct evaluation 3.464



c) ③ $A(w) = \int_3^w \sqrt{x-3}\,dx$

⑤ d) $\frac{dA}{dw}\Big|_{w=6} = ?$ ① question

using FTC ③ from part C

$\frac{dA}{dw} = \sqrt{w-3}\Big|_{w=6}$

$= \sqrt{3}$ ①

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④

$f(x) = x^3 - 6x^2 + P$

⑤ a) $f'(x) = \overset{①}{3x^2 - 12x}$

$3x(x-4) \overset{\text{set}}{=} 0$

$f' \quad + \quad 0 \quad - \quad 0 \quad +$ ①

$\qquad\qquad 0 \qquad\quad 4$

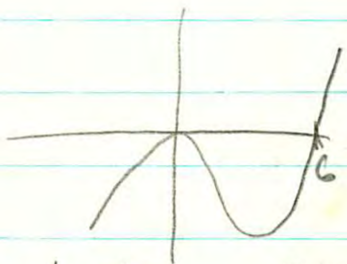
$f(0) = P \quad \overset{①}{(0, P)} \; \overset{①}{\text{max}} \; f' \text{ test}$

$f(4) = 64 - 96 + P$ ①

$(4, P-32)$ ① min f' test

⑥⑤ b) consider $g(x) = x^3 - 6x^2 + 0$

$x^2(x-6)$



If $-0 < P \overset{③}{<} 32$

③ including pict.

Then the "x-axis" will be between f(x) max and min values. Only then f will have 3 distinct roots. when $P = 0$ or $P = 32$ there will be one double root and another root, if $P > 0$ or if $P < 32$, there will be 1 real root and a conjugate pair of complex roots.

④ c) average value of fctN $\overset{①}{=} \dfrac{1}{b-a}\int_a^b f(x)\,dx$

$\dfrac{1}{2-(-1)} \displaystyle\int_{-1}^{2} x^3 - 6x^2 + P\,dx \overset{\text{set}}{=} 1$ ①

$\overset{①}{\dfrac{x^4}{4} - 2x^3 + Px}\Big|_{-1}^{2} = 3$

$4 - 16 + 2P - \left(\frac{1}{4} + 2 - P\right)$

$-12 + 2P - \frac{1}{4} - 2 + P$

$\to -14 - \frac{1}{4} + 3P = 3$

$3P = \frac{57}{4} + \frac{12}{4}$

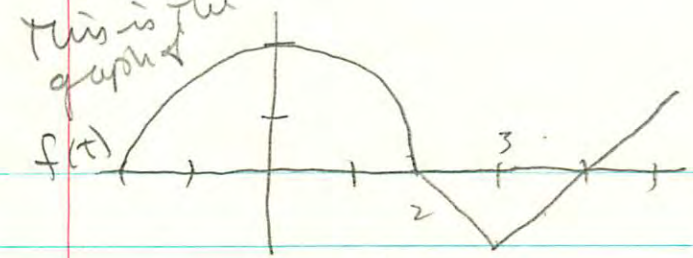
$3P = \frac{69}{4}$

① $P = \frac{23}{4}$

checked using Cal Tool Kit.
This average occurs when x = .972

1997
⑤

This is the graph of



$g(x) = \int_0^x f(t)\,dt$

④ a) $g(3) =$ ① $\int_0^3 f(t)\,dt$ = Area under $\frac{1}{4}$ circle radius $2 = \pi$ ②
Minus area of isosceles $\Delta = \frac{1}{2}$
Because it is Below axis.
$= \pi - \frac{1}{2}$ ① 3.642

③ b) g will have a max, min or PI at a critical value of x.
② This a zero of g'. That is when $f(t) = 0$
f is zero at −2, 2 and 4.

$$\begin{array}{cccc} 0 & + \quad 0 & - & 0 \quad + \\ -2 & 0 & 2 & 4 \end{array}$$

The only maximum occurs at ① $x = 2$ by f' test.

Note: This question is on the open interval $(-2, 5)$

③ c) tangent line at $x = 3$ has point $(3, \pi - \frac{1}{2})$ ①
from Part a.

$m = g'(3) = f(3) = -1$ ①
from graph

$y - y_1 = m(x - x_1)$
$y - (\pi - \frac{1}{2}) = -1(x - 3)$
$y = -x + 3 + \pi - \frac{1}{2}$
$y = -x + \frac{5}{2} + \pi$ ①
[illegible] 5.642✓

⑤ D) a point of inflection occurs when there is a
② change in concavity. g changes concavity when f' changes from increasing to decreasing. (or vice versa)

$$g'' = f' \quad \begin{array}{cccccc} & + & 0 & - \ \text{DNE} & 0 & + \\ -2 & & 0 & 2 & 3 & 4 \quad 5 \end{array}$$ ①

PI at $x = 0$ concave up to down
② $x = 3$ concave down to up

Note $f''(3)$ DNE (sharp corner) but existence of f'' not necessary to have PI

1997
⑥

$$\frac{dv}{dt} = -2v - 32 \qquad v(0) = -50$$

⑥ a) $\frac{dv}{dt} = -2(v+16)$

$$\int \frac{dv}{v+16} \overset{②}{=} -2\int dt$$

$$\ln|v+16| = -2t + C$$ ①

$$v+16 = e^{-2t+C}$$ ①

$$v+16 = Ae^{-2t}$$

$$v = Ae^{-2t} - 16$$

$$v(0) = A - 16 \overset{set}{=} -50$$

$$A = -34$$

$$v(t) = -34e^{-2t} - 16.$$ ②

③ b) $\lim_{t\to\infty} v(t) =$

$\lim_{t\to\infty} \frac{-34}{e^{2t}} - 16 =$

$0 - 16$

-16

Terminal velocity is -16 i.e. falling at 16 ft/sec.

⑥ c) $-34e^{-2t} - 16 \overset{\overset{①}{set}}{=} \overset{①}{-}20$

$$-34e^{-2t} = -4$$

$$\frac{17}{2} = e^{-2t}$$

② clean steps. $\ln\frac{17}{2} = -2t$

$$t = 1.070$$ ①

The skydiver will be falling at 20 ft/sec in 1.070 seconds. ① Sentence

Advanced Placement Program®
THE COLLEGE BOARD

1996 REPORT TO AP ® TEACHERS

MATHEMATICS - CALCULUS AB

VACAVILLE HIGH SCHOOL

The following charts indicate the number of AP candidates from your school/district in each fifth of the total candidate group for each category described.

MULTIPLE-CHOICE

		LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN
DIFFERENTIAL CALCULUS	(23 Q.)		3	2	5	8	23	9.4	13.6
INTEGRAL CALCULUS	(15 Q.)		1	2	6	9	15	6.0	9.1
PART A- NO CALCULATOR	(25 Q.)		1	2	8	7	25	11.0	16.7
PART B- WITH CALCULATOR	(15 Q.)	1	2	3	5	7	15	5.0	6.8
WEIGHTED M-C SCORE PERCENT IN EACH FIFTH			2 11%	2 11%	5 28%	9 50%	54	21.5	31.8
		LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN

AP GRADES	GRADE DISTRIBUTIONS	
	GLOBAL	GROUP
5	12.8%	50.0%
4	20.8%	11.1%
3	27.0%	33.3%
2	20.1%	5.6%
1	19.3%	0.0%
TOTAL CANDIDATES	105042	18

FREE-RESPONSE

	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN
QUESTION 1	2	2	4	5	5	9	4.0	5.3
QUESTION 2	2	3	1	7	5	9	5.2	6.1
QUESTION 3	1	1	2	7	7	9	2.5	3.9
QUESTION 4	1	1	2	5	9	9	3.0	4.4
QUESTION 5	1	2	2	3	10	9	1.7	3.5
QUESTION 6	2	2	7	3	4	9	2.5	2.9
WEIGHTED F-R SCORE PERCENT IN EACH FIFTH		1 6%	6 33%	2 11%	9 50%	54	18.9	26.1
	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN

AI053595
VACAVILLE HIGH SCHOOL
AP COORD/MS JEAN BROOKS
100 MONTE VISTA AVENUE
VACAVILLE CA 95688

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Advanced Placement Program®
THE COLLEGE BOARD

1996 REPORT TO AP ® TEACHERS

MATHEMATICS - CALCULUS AB

VACAVILLE HIGH SCHOOL

The following charts indicate the number of AP candidates from your school/district in each fifth of the total candidate group for each category described.

AP GRADES	GRADE DISTRIBUTIONS	
	GLOBAL	GROUP
5	12.8%	50.0%
4	20.8%	11.1%
3	27.0%	33.3%
2	20.1%	5.6%
1	19.3%	0.0%
TOTAL CANDIDATES	105042	18

MULTIPLE-CHOICE

	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN
DIFFERENTIAL CALCULUS (23 Q.)		3	2	5	8	23	9.4	13.6
INTEGRAL CALCULUS (15 Q.)		1	2	6	9	15	6.0	9.1
PART A- NO CALCULATOR (25 Q.)		1	2	8	7	25	11.0	16.7
PART B- WITH CALCULATOR (15 Q.)	1	2	3	5	7	15	5.0	6.8
WEIGHTED M-C SCORE PERCENT IN EACH FIFTH		2 11%	2 11%	5 28%	9 50%	54	21.5	31.8
	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN

FREE-RESPONSE

	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN
QUESTION 1	2	2	4	5	5	9	4.0	5.3
QUESTION 2	2	3	1	7	5	9	5.2	6.1
QUESTION 3	1	1	2	7	7	9	2.5	3.9
QUESTION 4	1	1	2	5	9	9	3.0	4.4
QUESTION 5	1	2	2	3	10	9	1.7	3.5
QUESTION 6	2	2	7	3	4	9	2.5	2.9
WEIGHTED F-R SCORE PERCENT IN EACH FIFTH		1 6%	6 33%	2 11%	9 50%	54	18.9	26.1
	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN

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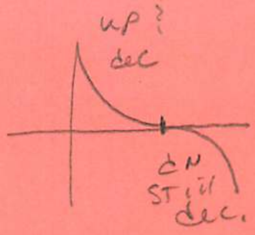
1996

① D)



② c) $\ln\frac{9}{4} = .811$

③ A) $C = 6,\ k = \frac{1}{5}\ln 2 = .139$

④ b) $x \in \left\{ \frac{-3\pi}{2}, \frac{\pi}{2} \right\}$ must have same sign
AND
must have same coordinate.

⑤ a) 150π cu ft. 471,239

⑥ b) $y = \frac{3}{5}x + 20$

2 correct +

1 correct
1 wrong ✓

0 correct
both tried 0

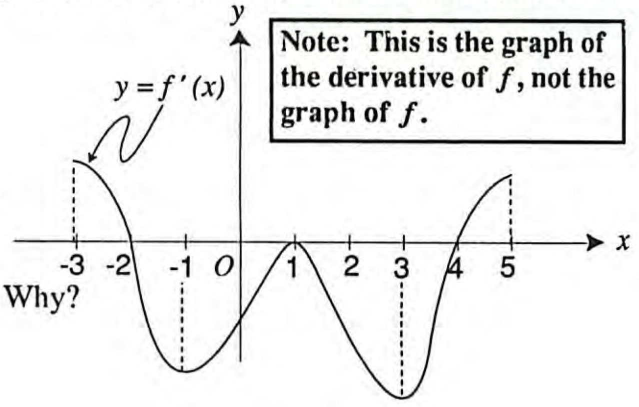
1 tried only
and wrong —

1 correct only & NOT tried

1996 Calculus AB

A graphing calculator is required for some problems or parts of problems on this section of the examination.

The figure on the right shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-3 \leq x \leq 5$.



Note: This is the graph of the derivative of f, not the graph of f.

a) For what values of x does f have a relative maximum? Why?

b) For what values of x does f have a relative minimum? Why?

c) On what intervals is the graph of f concave upward? Use f' to justify your answer.

d) Suppose that $f(1) = 0$. On an xy–plane, draw a sketch that shows the general shape of the graph of the the function f on the open interval $0 < x < 2$.

2. Let R be the region in the first quadrant under the graph of $y = \dfrac{1}{\sqrt{x}}$ for $4 \leq x \leq 9$.

a) Find the area of R.

b) If the line $x = k$ divides the region R into two regions of equal area, what is the value of k?

c) Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x-axis are squares.

3) The rate of consumption of cola in the United States is given by $S(t) = Ce^{kt}$, where S is measured in billions of gallons per year and t is measured in years from the beginning of 1980.

a) The consumption rate doubles every 5 years and the consumption rate is at the beginning of 1980 was 6 billion gallons per year. Find C and k.

b) Find the average rate of consumption of cola over the 10-year time period beginning January 1, 1983. Indicate units of measure.

c) Use the trapezoidal rule with four equal subdivisions to estimate $\int_5^7 S(t)\,dt$.

d) Using correct units, explain the meaning of $\int_5^7 S(t)\,dt$ in terms of colar consumption.

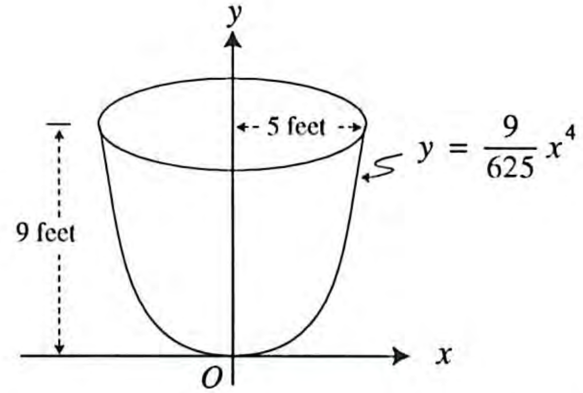
4) This problem deals with functions defined by $f(x) = x + b\sin x$, where b is a positive constant and $-2\pi \leq x \leq 2\pi$.

a) Sketch the graphs of two of these functions, $y = x + \sin x$ and $y = x + 3\sin x$ on the same axis.

b) Find the x-coordinates of all points, $-2\pi \leq x \leq 2\pi$, where the line $y = x + b$ is tangent to the graph of $f(x) = x + b\sin x$.

c) Are the points of tangency described in part (b) relative maximum points of f? Why?

d) For all values of $b > 0$, show that all inflection points of the graph of f lie on the line $y = x$.

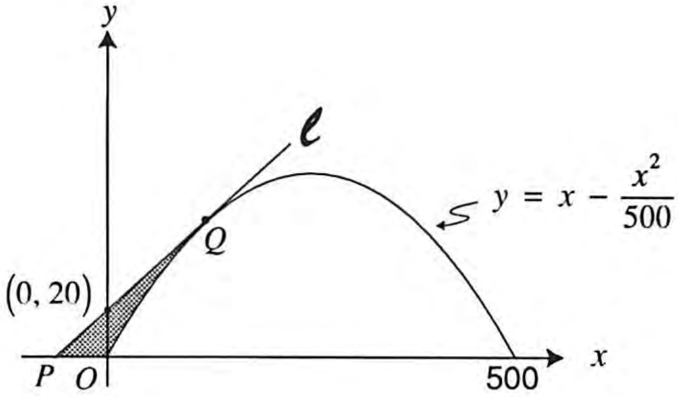


5) An oil storage tank has the shape shown above, obtained by revolving the curve $y = \frac{9}{625}x^4$ from $x = 0$ to $x = 5$ about the y-axis, where x and y are measured in feet. Oil flows into the tank at the constant rate of 8 cubic feet per minute.

a) Find the volume of the tank. Indicate units of measure.

b) To the nearest minute, how long would it take to fill the tank if the tank was empty initially?

c) Let h be the depth, in feet, of oil in the tank. How fast is the depth of the oil in the tank increasing when $h = 4$? Indicate units of measure.



6) Line ℓ is tangent to the graph of $y = x - \frac{x^2}{500}$ at the point Q , as shown in the figure above.

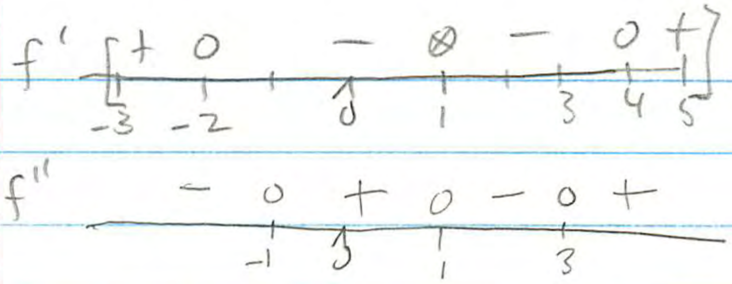
a) Find the x-coordinate of point Q .

b) Write an equation for line ℓ .

c) Suppose the graph of $y = x - \frac{x^2}{500}$ shown in the figure, where x and y are measured in feet, represents a hil. Thre is a 50-foot tree growing vertically at the top of the hill. Does a spotlight at point P directed along the line ℓ shine on any part of the tree? Show the work that leads to your conclusion.

1996

1996
①


f'
+ 0 − ⊗ − 0 +
-3
-2
0
1
3
4
5
f''
− 0 + 0 − 0 +
-1
0
1
3

④ a) f has rel max $x = -2$ 1pT f' test 1pT
$x = 5$ ENDPT f increases to it.
1pT

④ b) f has rel min 1pT $x = -3$ ENDPT. 1pT f increase from it.
$x = 4$ f' test 1pT
1pT

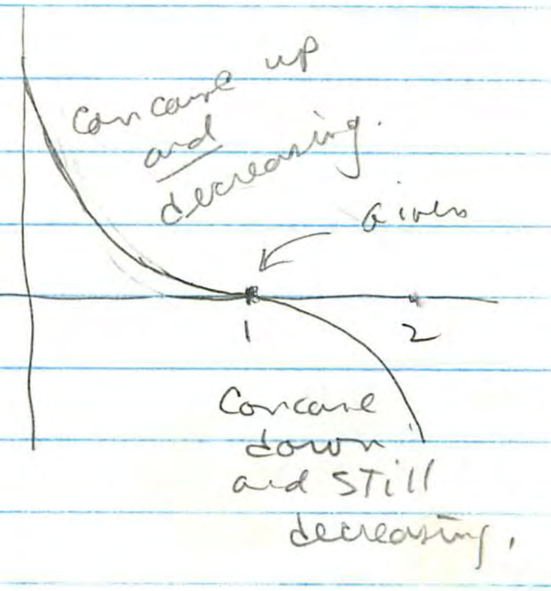
④ c) f'' is 1st der of f'. when f' is increasing f'' is > 0
when f' is decreasing f'' is < 0
f is concave up when $f'' > 0$
f is concave down when $f'' < 0$
f' used to create f'' # line.

used f'' to justify

f is concave up for ① $-1 < x < 1$ and when $3 < x < 5$ ①

③ D) $f(1) = 0$ $0 < x < 2$

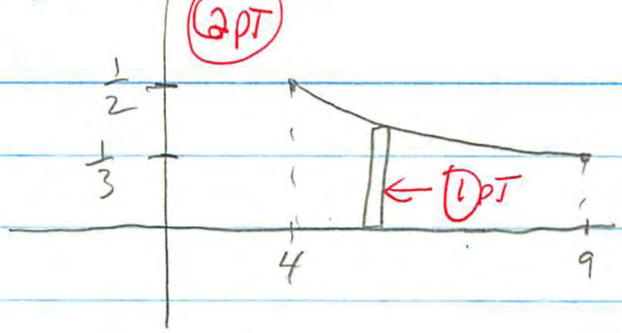
Picture 2pts
ANNOTATION 1pT


Concave up and decreasing.
given
1
2
Concave down and STill decreasing.

1996

②

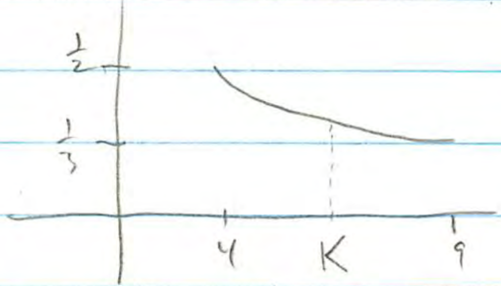
$Y = \frac{1}{\sqrt{x}} \quad 4 \le x \le 9$



⑦ a) $\int_4^9 \frac{1}{\sqrt{x}}\,dx =$ 2 pTS

$2\sqrt{x}\Big|_4^9 = 2\cdot3 - 2\cdot2 \overset{①}{=} 2$ sq units $\overset{①}{=}$ area of R,

④ b) $\int_4^K \frac{1}{\sqrt{x}}\,dx \overset{set}{=} \int_K^9 \frac{1}{\sqrt{x}}\,dx$ ②



$2\sqrt{x}\Big|_4^K = 2\sqrt{x}\Big|_K^9$ OR $\int_4^K \frac{1}{\sqrt{x}}\,dx \overset{set}{=} 1$ which is $\frac{1}{2}$ of area fm Part "a".

$2\sqrt{K} - 4 = 2\cdot3 - 2\sqrt{K}$

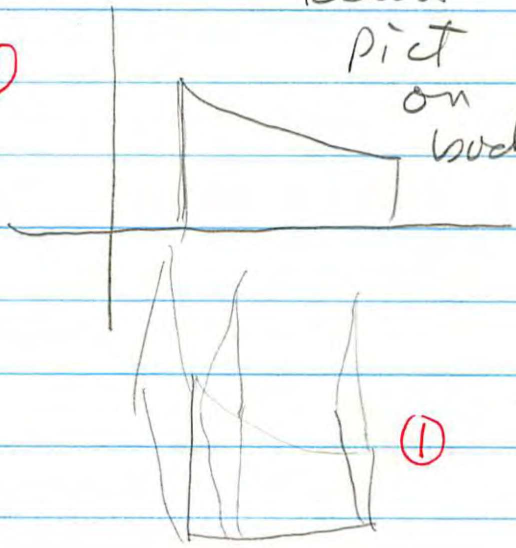
$4\sqrt{K} = 10$

$\sqrt{K} = \frac{5}{2}$

$K = \frac{25}{4}$ ①

when $K = \frac{25}{4}$ ① R is divided into 2 equal areas.

Better pict on back,



④ c) base is R.

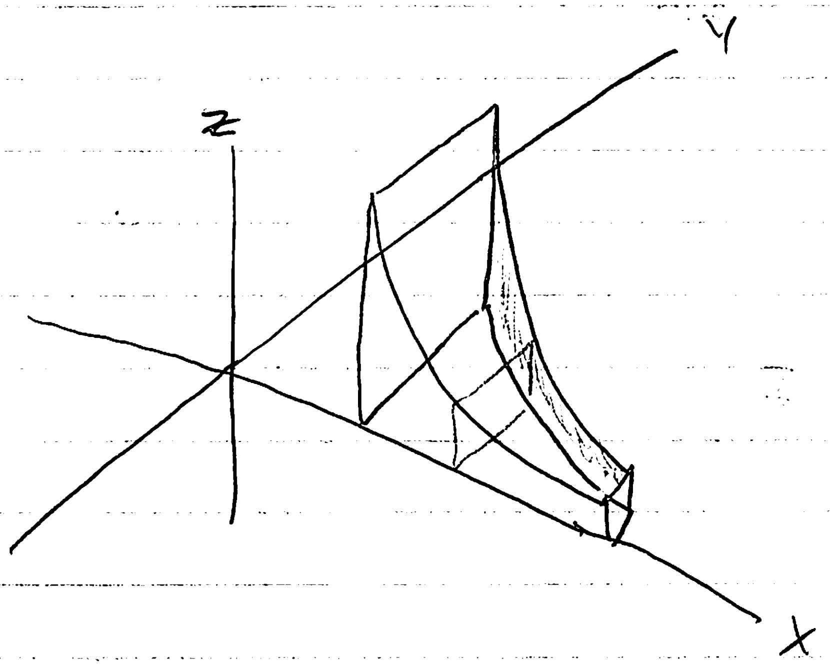
Sum up squares.

Side of square is $\frac{1}{\sqrt{x}}$ ∴

area of square is $\frac{1}{x}$

Sum of squares: $\int_4^9 \frac{1}{x}\,dx$ ② $= \ln x\Big|_4^9 = \ln 9 - \ln 4$

$\ln\frac{9}{4}$ ① volume of solid. $\Rightarrow .811$


z
y
x

1996
③

$S(t) = Ce^{kT}$ $\quad T=0 \Rightarrow 1980$

③ a) $2c \overset{set}{=} ce^{(k)(5)}$ $\quad S(0)=6$ $\quad S(t) = 6e^{\frac{T}{5}\ln 2}$

$2 = e^{5k}$ $\quad 6 \overset{set}{=} c \cdot 1$

$2^{\frac{1}{5}} = e^{k}$ $\quad \therefore c = 6$ ① $\quad = 6(2)^{\frac{T}{5}}$

$\ln 2^{\frac{1}{5}} = k$ ② $\doteq .139$

⑥ b) average consumption. ① $\frac{1}{b-a}\int_a^b f(x)\,dx$ $\quad \frac{1}{10}\int_{3}^{13} 6e^{\frac{T}{5}\ln 2}\,dt$ ① ① ①

$\frac{3}{5}\int_3^{13} e^{\frac{T}{5}\ln 2}\,dt = \frac{3}{5}\cdot e^{\frac{T}{5}\ln 2}\cdot\frac{1}{\frac{1}{5}\ln 2}\Big|_3^{13}$

$\frac{3}{\cancel{5}}\cdot 2^{\frac{T}{5}}\cdot\frac{\cancel{5}}{\ln 2} = \frac{3\,2^{\frac{T}{5}}}{\ln 2}\Big|_3^{13} = \frac{3}{\ln 2}\left[2^{\frac{13}{5}} - 2^{\frac{3}{5}}\right]$

$\frac{3}{\ln 2}2^{\frac{3}{5}}(2^2-1)$

$\frac{9}{\ln 2}\cdot 2^{\frac{3}{5}}$

19.675

① 19.680 Billion gallons of cola

① was consumed on average
Per year during the years 1983 to 1993.

④ c) trapezoid rule. $\frac{h}{2}\left(f(x_0)+2f(x_1)+\dots+2f(x_{n-1})+f(x_n)\right)$

4 subdivision $\frac{7-5}{4} = \frac{1}{2} = h$

$\frac{1}{2}\cdot\frac{1}{2}(A+2B+2C+2D+E)$

$\frac{1}{4}(\qquad) = 27.68$

3pts Shows method.

27.68 ①

27.666

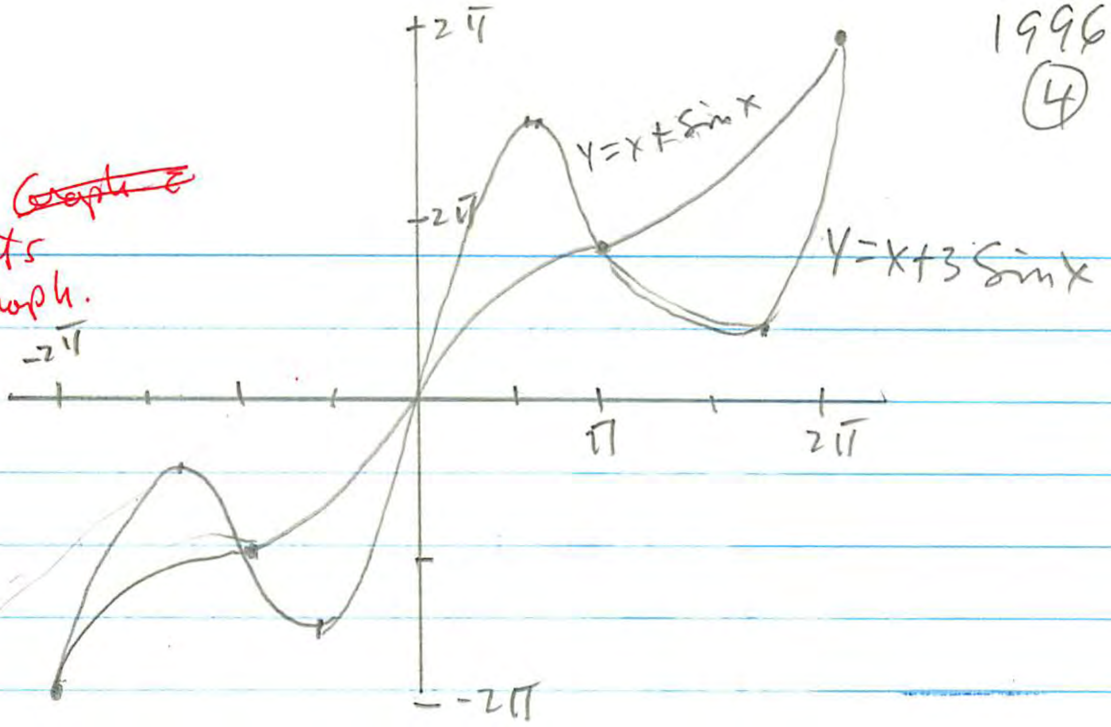
x	f(x)	
$x_0 = 5$	12.0	A
$x_1 = 5.5$	12.861	B
$x_2 = 6$	13.784	C
$x_3 = 6.5$	14.774	D
$x_4 = 7$	15.834	E

② D) During the years of 1985 and 1986 about 17.667 Billion gallons of cola were consumed.

1996

④

~~Graph 8~~
No points for graph.



⑦ b) $y = x + b$ $f(x) = x + b\sin x$

② → line is Tangent when has same slope AND shares coordinate same point.

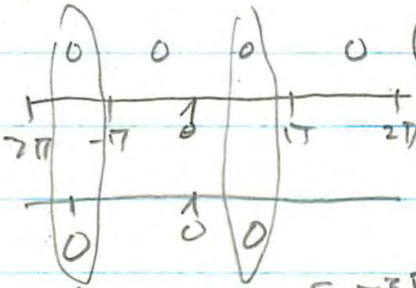
same slope

$y' = 1$ $f'(x) = 1 + b\cos x$

② $1 \overset{set}{=} 1 + b\cos x$

$b\cos x = 0$

$x = \{-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}\}$



same point.

$x + b \overset{set}{=} x + b\sin x$

② $b - b\sin x = 0$

$b(1 - \sin x) = 0$

$x = \{-\frac{3\pi}{2}, \frac{\pi}{2}\}$

both are satisfied when $x = \{-\frac{3\pi}{2}, \frac{\pi}{2}\}$ ①

③ c) A point is an extreme value if it is a critical value ($f'(x) = 0$), an end point or a place where the function is continuous but the derivative does not exist (cusp)

1 for correct conclusions
2 proper reason

The points in (b) are not critical values $f'(-\frac{3\pi}{2}) = f'(\frac{\pi}{2}) = 1$ not end PTs and the derivative does exist therefore the points in (b) are not relative maximum points, more ↓

1986

④

continued

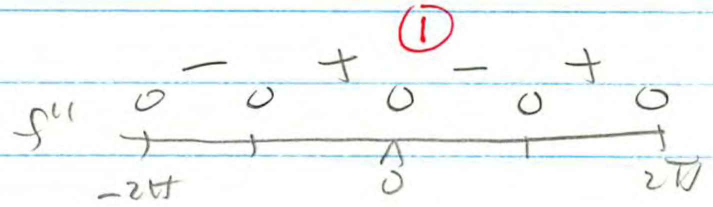
4

D

⑤ $f(x) = x + b\sin x$

$f'(x) = 1 + b\cos x$

$f''(x) = -b\sin x$



$-b\sin x \overset{Set}{=} 0$

$x \in \{k\pi\}$ $f''(\frac{\pi}{2}) = -b(+1) < 0$

① f has PI when f changes concavity. this occurs when f'' changes signs

① f has PI at $-\pi$, 0, π because f changes concavity at those x values.

2pts

$f(-\pi) = -\pi + 0 = -\pi$ $(-\pi, -\pi)$

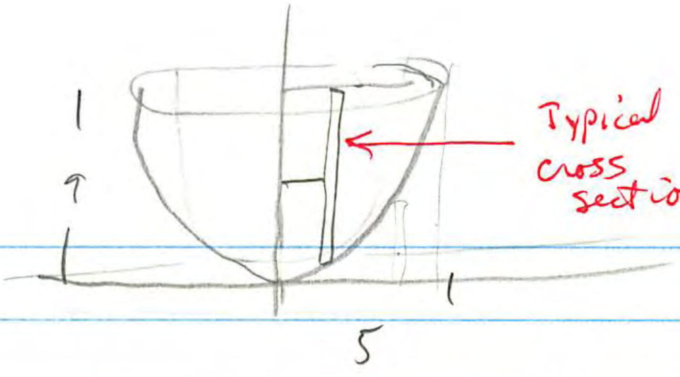
$f(0) = 0 + 0 = 0$ $(0, 0)$

$f(\pi) = \pi + 0 = \pi$ (π, π) are

The coordinates of the PI which also satisfy $y = x$.

There are no other PI (so none lie off of $y = x$)

1996
⑤
Shells



Typical cross section 1pT.

⑦ a) Volume? "Shells" ① $2\pi\int r$ height

③ $$2\pi\int_0^5 x\left(9-\frac{9}{625}x^4\right)dx$$

$$18\pi\int_0^5 x-\frac{x^5}{625}\,dx$$

$$18\pi\left(\frac{x^2}{2}-\frac{x^6}{625\cdot 6}\right|_0^5$$

$$18\pi\left(\frac{25}{2}-\frac{x^4\cdot x^2}{\cancel{625}\ 6}\right.$$

$$\left(\frac{25}{2}-\frac{25}{6}\right.$$

$$\overset{3}{\cancel{18}}\pi\frac{50}{\cancel{6}}$$

150π cu feet.

②pts

② b) $\frac{150\pi}{8}$ ① ≈ 58.9

The tank could be filled in ① about 59 minutes.

c) on Next Page.

4) Shells

$$v = 2\pi \int_0^x 9\tau \, d\tau - \frac{18\pi}{625}\int_0^x \tau^5 \, d\tau$$

$$v = \cancel{2}\pi \frac{9x^2}{\cancel{2}} - \frac{\overset{3}{\cancel{18}}\pi}{625}\frac{\overset{5^6}{x^6}}{\cancel{6}} \Big|_0^5$$

$$225\pi - 3\cdot 25\pi$$

$$150\pi. \quad \checkmark$$

checking....

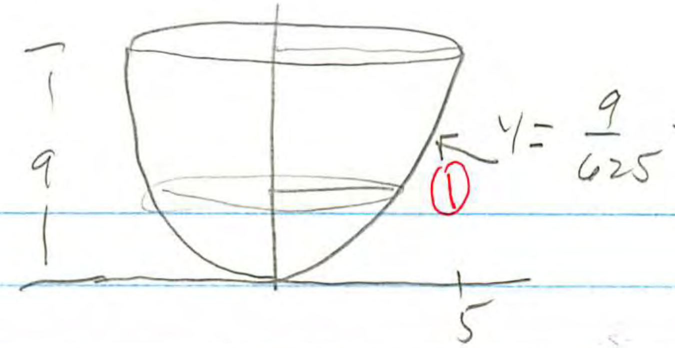
$$\frac{dv}{dt} = \left(2\pi \cdot 9x - \frac{18\pi}{625}x^5\right)\frac{dx}{dt}$$

$$= 2\pi x\left(9 - \frac{9}{625}x^4\right)\frac{dx}{dt}$$

which is the same as when using other method.

1996
⑤

DISKS



⑦ a) volume using "disks" ① $\pi\int r^2\,dy$

using "shells" on back of this page $\frac{150\pi}{1}$

$$V = \pi\int_0^9 \frac{25}{3}y^{\frac{1}{2}}\,dy$$

$$\frac{625}{9}y = x^4$$

$$\frac{5}{\sqrt{3}}y^{\frac{1}{4}} = x$$

$$= \frac{25\pi}{3}\,y^{\frac{3}{2}}\cdot\frac{2}{3}\Big|_0^9$$

$$V = \frac{25\pi}{\cancel{3}}\cdot\overset{3}{\cancel{27}}\cdot\frac{2}{\cancel{3}}$$

$$V = 150\pi$$

The volume of the tank is 150π cu gallons

② b) $\frac{150\pi}{8} \approx 58.9$ ①

The tank could fill ① in about 59 minutes.

FTC $\frac{dv}{dt} = \pi\left(\frac{25}{3}\sqrt{h}\right)\frac{dh}{dt}$

⑥ c) $V = \pi\int_0^h \frac{25}{3}y^{\frac{1}{2}}\,dy$

$\frac{dv}{dt} = 8$

$\frac{dh}{dt} = ?$

$$V = \frac{25\pi}{3}\,\frac{2}{3}\,y^{\frac{3}{2}}\Big|_0^h = \frac{50\pi h^{3/2}}{9}$$

Relationship between v and h ②

$$\frac{dv}{dt} = \frac{\overset{25}{\cancel{50}}\pi}{\cancel{9}\,3}\cdot\frac{\cancel{3}}{\cancel{2}}h^{\frac{1}{2}}\frac{dh}{dt}$$

Proper der ②

$$8 = \frac{25\pi}{3}\cdot 2\,\frac{dh}{dt}$$

$$\frac{12}{25\pi} = \frac{24}{50\pi} = \frac{dh}{dt}$$ ①

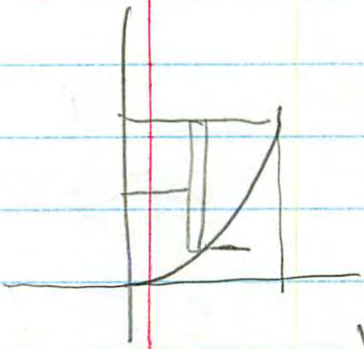
The oil height ① is increasing at $\frac{12}{25\pi}$ inches/minute

$y = \frac{9}{625}x^4 \Rightarrow x = \overline{5}$

$\frac{625}{9}y = x^4$

$\frac{5y}{3^{\frac{1}{2}}} = x$

a) "Shells"



$$2\pi\int_0^5 x\left[9 - \frac{9}{625}x^4\right]dx$$

$$\frac{18\pi}{625}\int_0^5 625x - x^5\,dx$$

$$\frac{18\pi}{625}\left[\frac{625x^2}{2} - \frac{x^6}{6}\right|_0^5$$

$2\pi\cdot 25$

$625\cdot 5^2$

$\frac{5^6}{2} - \frac{5^6}{6}$

$$\frac{\overset{3}{\cancel{18}}\pi\cdot}{\cancel{6}\;5^4}\left[3\cdot5^6 - 5^6\right]$$

$3\pi\cdot\;\cdot5^2[2]$

$6\cdot25\pi$

150π

⑥ c)

1996
⑤
continued.

Given $\frac{dv}{dt}=8$ $\frac{dh}{dt}=?$

height is y value ∴

①

$$h = \frac{9}{625}x^4$$

$$\frac{625}{9}h = x^4$$

$$\frac{5}{\sqrt{3}}h^{\frac{1}{4}} = x$$

$$\frac{5}{4\sqrt{3}}\cdot h^{-\frac{3}{4}}\frac{dh}{dt} = \frac{dx}{dt}$$

Volume for ANY x value: $v = 18\pi\int_0^x \tau - \overset{①}{} \frac{\tau^5}{625}\,d\tau$

Volume in terms of x $v = 18\pi\left[\frac{x^2}{2} - \frac{x^6}{625\cdot 6}\right]$

① $\frac{dv}{dt} = 18\pi\left[x - \frac{x^5}{625}\right]\overset{①}{\frac{dx}{dt}}$

$$2\pi x\left[9 - \frac{9x^4}{625}\right]$$

$$\frac{dv}{dt} = 2\pi\left(\frac{5}{\sqrt{3}}h^{\frac{1}{4}}\right)\left[9-h\right]\frac{5}{4\sqrt{3}}\frac{1}{h^{\frac{3}{4}}}\frac{dh}{dt}$$

$$8 = \cancel{2}\pi\cdot\frac{5\cdot\cancel{\sqrt{2}}}{\sqrt{3}}\left[\frac{5}{1}\right]\frac{5}{4\sqrt{3}}\frac{1}{\underset{\cancel{2}}{(\sqrt{2})^3}}\frac{dh}{dt}$$

$$8 = \frac{125\pi}{12}\frac{dh}{dt}$$

① $\frac{96}{125\pi} = \frac{dh}{dt} \approx .244$ The depth of the oil is rising ①
at $\frac{96}{125\pi}$ inches per minute.
when the depth is 4 feet.

96

5c

"Shells." $h = \frac{9}{625}x^4$ $\frac{625h}{9} = x^4$

from part a "shells" $\frac{5h^{\frac{1}{4}}}{3^{\frac{1}{2}}}$ $\frac{5h^{\frac{1}{4}}}{3^{\frac{1}{2}}} = x$

$$V = 2\pi \int_0^{\frac{5h^{1/4}}{3^{1/2}}} x\left[9 - \frac{9}{625}x^4\right] dx$$

$$= \frac{18\pi}{625}\left[\frac{625}{2}x^2 - \frac{x^6}{6}\right|_0^{\frac{5h^{1/4}}{3^{1/2}}}$$

$$= \frac{18\pi}{625}\left[\frac{625}{2}\cdot\frac{25h^2}{3} - \frac{5^6 h^{\frac{3}{2}}}{6\cdot 27}\right]$$

$$25\,\frac{\overset{3}{\cancel{18}}\pi}{\cancel{625}}\cdot\frac{\cancel{625}}{\underset{1}{\cancel{6}}}\left[h^2 - \frac{h^{\frac{3}{2}}}{27}\right]$$

$$V = 75\pi\left[h^2 - \frac{h^{\frac{3}{2}}}{27}\right]$$

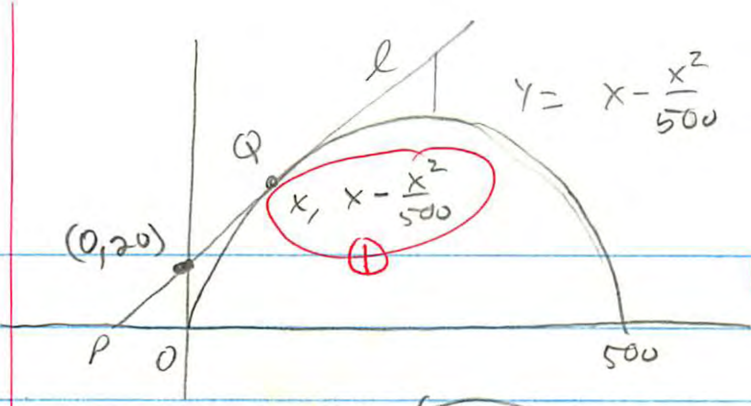
$$\frac{dV}{dt} = 75\pi\left(2h - \frac{3}{2}h^{\frac{1}{2}}\cdot\frac{1}{27}\right)\frac{dh}{dt}\Big|_{h=4}$$

$$\frac{8}{75\pi} = \left(8 - \frac{1}{9}\right)\frac{dh}{dt}$$

$$\frac{71}{9}$$

$$\frac{\cancel{9}}{71}\,\frac{8}{\underset{25}{\cancel{75}}\pi} =$$

1996
⑥



⑥ a) Slope of line $\overset{\text{Set}}{=}$ y' ①

$$\frac{x - \frac{x^2}{500} - 20}{x} = 1 - \frac{x}{250}$$ ①

$$\frac{500x - x^2 - 10000}{500x} = \frac{250 - x}{250}$$ ①

$\cancel{500x} - x^2 - 10000 = \cancel{500x} - 2x^2$

$x^2 = 10,000$

① $x = 100 \quad (x > 0)$

The x-coordinate of Q is 100. ①

③ b) Slope of line is $y'|_{x=100} = 1 - \frac{100}{250}$ ①

$1 - \frac{2}{5} = \frac{3}{5} = m$

y intercept is 20 $\therefore$ $y = \frac{3}{5}x + 20$ is the line ℓ ②

⑥ c) tree top is max of y + 50 feet.

description of method ①

If line evaluated at x value of max of y is ≤ max y + 50 feet then light shines on tree.

$y = x - \frac{x^2}{500}$ $\qquad y|_{x=250} = 250 - \frac{\overset{125}{\cancel{250 \cdot 250}}}{\underset{2}{\cancel{500}}}$

$y' = 1 - \frac{x}{250} \overset{\text{set}}{=} 0$

$y'' = \frac{-1}{250} < 0$ $\qquad x = 250 \qquad y_{max} = 125$ ①

max f'' test.

Max to top of tree is $125 + 50 = 175$ feet. ①

$\text{line}|_{x=250} \rightarrow \quad y = \frac{3}{5}(\overset{50}{\cancel{250}}) + 20$ ①

$y = 170$ ①

since $170 \leq 175$, the light does hit the tree (5 feet from the top). ①

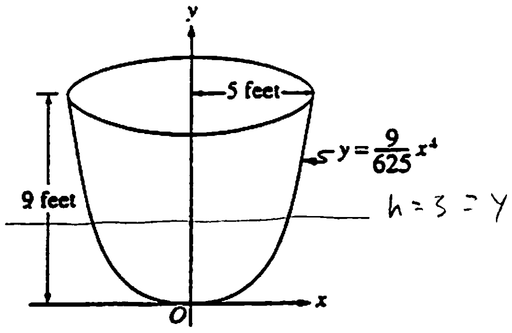
AB-5

5. An oil storage tank has the shape shown above, obtained by revolving the curve $y = \frac{9}{625}x^4$ from $x = 0$ to $x = 5$ about the y-axis, where x and y are measured in feet. Oil flows into the tank at the constant rate of 8 cubic feet per minute.

   (a) Find the volume of the tank. Indicate units of measure.

   (b) To the nearest minute, how long would it take to fill the tank if the tank was empty initially?

   (c) Let h be the depth, in feet, of oil in the tank. How fast is the depth of the oil in the tank increasing when $h = 4$? Indicate units of measure.



(a) $\text{volume} = V = \pi\int_0^9 \frac{25}{3}\sqrt{y}\,dy = 150\pi \text{ ft}^3$ (or 471.238 ft^3 or 471.239 ft^3) $V = 2\pi\int_0^5 x\left(9 - \frac{9}{625}x^4\right)dx$	$3\begin{cases} 2:\text{integral}\begin{cases}1:\text{limits and }\pi/2\pi\\ \quad\pi\int_0^9(\,)\,dy,\quad 2\pi\int_0^5(\,)\,dx\\ 1:\text{integrand}\end{cases}\\ 1:\text{answer with units}\end{cases}$ not eligible for answer point unless first two points are earned.
(b) $\text{time} = \frac{\text{volume}}{\text{rate}} = \frac{150\pi}{8}$ therefore, 59 minutes	1: integer answer
(c) $V = \pi\int_0^h \frac{25}{3}\sqrt{y}\,dy$ $\frac{dV}{dt} = \frac{25}{3}\pi\sqrt{h}\frac{dh}{dt}$ $\frac{dV}{dt} = 8$ when $h = 4$, $8 = \frac{25}{3}\pi(2)\frac{dh}{dt}$ $\frac{dh}{dt} = \frac{12}{25\pi}$ ft/min (or 0.152 ft/min or 0.153 ft/min)	$5\begin{cases}1:\text{volume as definite integral using } h\\ 2:\text{finding }\frac{dV}{dt}\begin{cases}1:\frac{dV}{dh}\\ 1:\frac{dV}{dt}=\frac{dV}{dh}\cdot\frac{dh}{dt}\end{cases}\\ 1:\frac{dV}{dt}=8\\ 1:\text{answer with units}\end{cases}$ if V is linear, max 2/5 (0 - 0 - 1 - 1 - 0) if V is constant, max 1/5 (0 - 0 - 0 - 1 - 0)

QUESTION #6 / STANDARDS

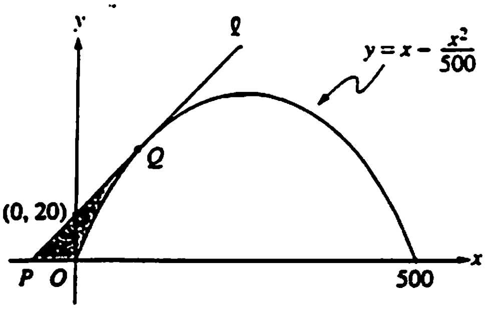
AB-6

6. Line ℓ is tangent to the graph of $y = x - \frac{x^2}{500}$ at the point Q, as shown in the figure above.

(a) Find the x-coordinate of point Q.

(b) Write an equation for line ℓ.

(c) Suppose the graph of $y = x - \frac{x^2}{500}$ shown in the figure, where x and y are measured in feet, represents a hill. There is a 50-foot tree growing vertically at the top of the hill. Does a spotlight at point P directed along line ℓ shine on any part of the tree? Show the work that leads to your answer.



(a) let Q be $\left(a, a - \frac{a^2}{500}\right)$.

$$\frac{dy}{dx} = 1 - \frac{x}{250}$$

setting slopes equal:

$$1 - \frac{a}{250} = \frac{\left(a - \frac{a^2}{500}\right) - 20}{a}$$

$$a = 100$$

or

$$\frac{dy}{dx} = 1 - \frac{x}{250}$$

equation for ℓ: $y = \left(1 - \frac{a}{250}\right)x + 20$

setting y-values equal:

$$\left(1 - \frac{a}{250}\right)a + 20 = a - \frac{a^2}{500}$$

$$a = 100$$

4
- 1 : slope of tangent line from parabola
- 1 : uses the condition that $\left(a, a - \frac{a^2}{500}\right)$ is on line ℓ
- 1 : uses the condition that slopes are equal at Q
- 1 : answer
  - 0/1 if student is solving an irrelevant equation

(b) $y = \frac{3}{5}x + 20$

2
- 1 : slope
  - 0/1 if $m \le 0$
- 1 : equation

(c) height of hill at $x = 250$:

$$y = 250 - \frac{250^2}{500} = 125 \text{ feet}$$

height of line at $x = 250$:

$$y = \frac{3}{5}(250) + 20 = 170 \text{ feet}$$

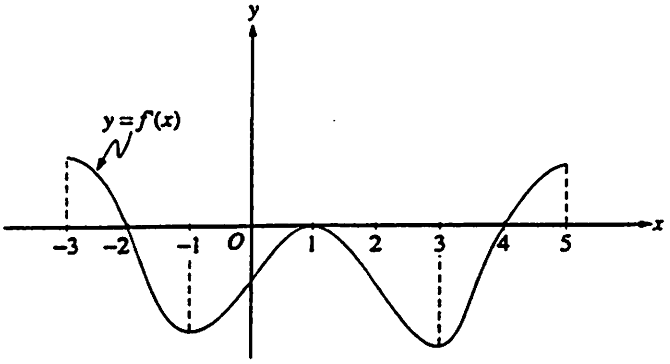
Yes, the spotlight hits the tree since the height of the line is less than the height of the hill + tree which is 175 feet.

3
- 1 : height of hill
- 1 : height of line
  - 0/1 if height < 0
- 1 : answer with analysis

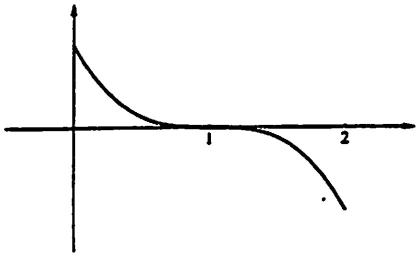
96

AB-1

1. The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-3 < x < 5$.
   (a) For what values of x does f have a relative maximum? Why?
   (b) For what values of x does f have a relative minimum? Why?
   (c) On what intervals is the graph of f concave upward? Use f' to justify your answer.
   (d) Suppose that $f(1) = 0$. In the xy-plane provided, draw a sketch that shows the general shape of the graph of the function f on the open interval $0 < x < 2$.



Note: This is the graph of the derivative of f, not the graph of f.

(a)	$x = -2$ $f'(x)$ changes from positive to negative at $x = -2$, or f is increasing to the left of $x = -2$ and decreasing to the right of $x = -2$.	$2\begin{cases}1: x = -2\\ 1: \text{reason}\end{cases}$
(b)	$x = 4$ $f'(x)$ changes from negative to positive at $x = 4$, or f is decreasing to the left of $x = 4$ and increasing to the right of $x = 4$.	$2\begin{cases}1: x = 4\\ 1: \text{reason}\end{cases}$
(c)	$(-1, 1)$ and $(3, 5)$ f' is increasing on these intervals.	$3\begin{cases}1: (-1,1)\\ 1: (3,5)\\ 1: \text{reason using } f'\end{cases}$ max 1/3 for one incorrect interval 0/3 for two incorrect intervals
(d)		$2\begin{cases}<-1>\ f(1) \neq 0\\ <-1>\ \text{not decreasing}\\ <-1>\ \text{incorrect concavity}\\ <-1>\ f'(1) \neq 0\end{cases}$

QUESTION #2 / STANDARDS

AB-2

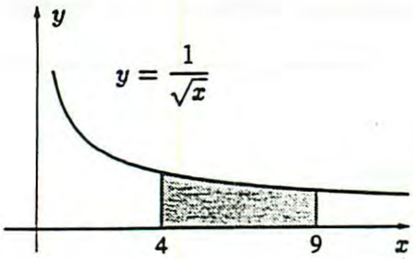
2. Let R be the region in the first quadrant under the graph of $y = \dfrac{1}{\sqrt{x}}$ for $4 \le x \le 9$.

   (a) Find the area of R.

   (b) If the line $x = k$ divides the region R into two regions of equal area, what is the value of k?

   (c) Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x-axis are squares.

(a) $\displaystyle\int_4^9 \frac{dx}{\sqrt{x}} = 2$



$$3 \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \\ \quad 0/1 \text{ if integrand is incorrect} \end{cases}$$

(b) $\displaystyle\int_4^k \frac{dx}{\sqrt{x}} = 1$

SOLVE(fnInt(1/√X,X,4,X)−1,X,6)

$$2\sqrt{x}\Big|_4^k = 1$$

$$2\sqrt{k} - 2\sqrt{4} = 1$$

$$k = \frac{25}{4}$$

$$\left(\text{or } \int_k^9 \frac{dx}{\sqrt{x}} = 1 \text{ or } \int_4^k \frac{dx}{\sqrt{x}} = \int_k^9 \frac{dx}{\sqrt{x}}\right)$$

$$3 \begin{cases} 1 : \displaystyle\int_4^k \frac{dx}{\sqrt{x}} \text{ or } \int_k^9 \frac{dx}{\sqrt{x}} \\ 1 : \text{equation involving the two halves of } R \\ 1 : \text{answer} \\ \quad 0/1 \text{ if answer from equation not involving relevant areas} \end{cases}$$

(c) $$\text{volume} = \int_4^9 \left(\frac{1}{\sqrt{x}}\right)^2 dx$$

$$= \int_4^9 \frac{dx}{x} = \ln x\Big|_4^9 = \ln\frac{9}{4}$$

(or 0.811)

$$3 \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \\ \quad 0/1 \text{ if integrand is incorrect} \end{cases}$$

QUESTION #3 / STANDARDS

AB-3, BC-3

3. The rate of consumption of cola in the United States is given by $S(t) = Ce^{kt}$, where S is measured in billions of gallons per year and t is measured in years from the beginning of 1980.

   (a) The consumption rate doubles every 5 years and the consumption rate at the beginning of 1980 was 6 billion gallons per year. Find C and k.

   (b) Find the average rate of consumption of cola over the 10-year time period beginning January 1, 1983. Indicate units of measure.

   (c) Use the trapezoidal rule with four equal subdivisions to estimate $\int_5^7 S(t)\,dt$.

   (d) Using correct units, explain the meaning of $\int_5^7 S(t)\,dt$ in terms of cola consumption.

(a) $S(t) = Ce^{kt}$

$$S(0) = 6 \implies C = 6$$

$$S(5) = 12 \implies 12 = 6e^{5k}$$

$$2 = e^{5k}$$

$$k = \frac{\ln 2}{5} \quad (0.138 \text{ or } 0.139)$$

$$3\begin{cases} 1: C = 6 \\ 1: 12 = 6e^{5k} \\ 1: k = \dfrac{\ln 2}{5} \end{cases}$$

(b) average rate $= \dfrac{1}{13-3}\displaystyle\int_3^{13} 6e^{\frac{\ln 2}{5}t}\,dt$

$= \dfrac{3}{\ln 2}\left[e^{2.6\ln 2} - e^{0.6\ln 2}\right]$ billion gal/yr

(19.680 billion gal/yr)

$$3\begin{cases} 1: \text{uses } [3,13] \text{ and divides by } 13-3 \\ 1: \text{integrand} \\ 1: \text{answer with units} \\ \quad 0/1 \text{ if not } \dfrac{1}{b-a}\displaystyle\int_a^b S(t)\,dt \end{cases}$$

(c) $\displaystyle\int_5^7 S(t)\,dt$

$\doteq \dfrac{1}{4}[S(5) + 2S(5.5) + 2S(6) + 2S(6.5) + S(7)]$

$$1\begin{cases} \text{Trapezoidal rule with } S, \\ n = 4, \text{ interval } [5,7] \end{cases}$$

(d) This gives the total consumption, in billions of gallons, during the years 1985 and 1986.

$$2\begin{cases} 1: \text{total consumption in a time period} \\ 1: \begin{cases} \text{correct time period} \\ \text{liquid measure} \end{cases} \end{cases}$$

0/2 for "rate of consumption"

QUESTION #4 / STANDARDS

AB-4, BC-4

4. This problem deals with functions defined by $f(x) = x + b\sin x$, where b is a positive constant and $-2\pi \le x \le 2\pi$.

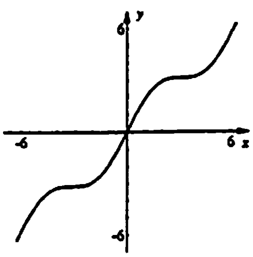
   (a) Sketch the graphs of two of these functions, $y = x + \sin x$ and $y = x + 3\sin x$, as indicated below.

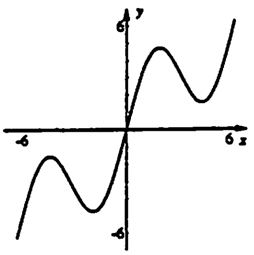
   (b) Find the x-coordinates of all points, $-2\pi \le x \le 2\pi$, where the line $y = x + b$ is tangent to the graph of $f(x) = x + b\sin x$.

   (c) Are the points of tangency described in part (b) relative maximum points of f? Why?

   (d) For all values of $b > 0$, show that all inflection points of the graph of f lie on the line $y = x$.

(a)





$$2\begin{cases} 1: \text{graph of } y = x + \sin x \\ \quad (\text{increasing, through origin,} \\ \quad \text{cd - cu - cd - cu}) \\ 1: \text{graph of } y = x + 3\sin x \\ \quad (\text{inc - dec - inc - dec - inc,} \\ \quad \text{through origin,} \\ \quad \text{cd - cu - cd - cu}) \end{cases}$$

(b) $y' = 1 = 1 + b\cos x$

$b\cos x = 0$

$\cos x = 0$

$y = x + b = x + b\sin x$

$b = b\sin x$

$1 = \sin x$

$$x = -\frac{3\pi}{2} \text{ or } \frac{\pi}{2}$$

$$4\begin{cases} \text{1: derivatives of } x + b \text{ and } x + b\sin x \\ \text{1: sets derivatives equal} \\ \text{1: sets } y\text{-values equal} \\ \text{1: answer from merged information} \end{cases}$$

max: 0 - 1 - 1 - 0 if specific value(s) for b

max: 0 - 0 - 1 - 0 if no derivative

(c) No, because $f'(x) = 1$ (or $f'(x) \neq 0$) at x-coordinates of points of tangency.

1: answer with reason

(d) $f''(x) = -b\sin x = 0$

$\sin x = 0$

$f(x) = x + b \cdot 0 = x$
at x-coordinates of any inflection points

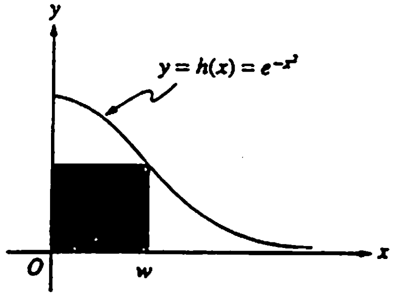
$$2\begin{cases} \text{1: sets second derivative equal to 0} \\ \text{1: shows } f(x) = x \end{cases}$$

0/2 if specific value(s) for b

QUESTION #1 / STANDARDS

BC-1

1. Consider the graph of the function h given by $h(x) = e^{-x^2}$ for $0 \le x < \infty$.
   (a) Let R be the unbounded region in the first quadrant below the graph of h. Find the volume of the solid generated when R is revolved about the y-axis.
   (b) Let $A(w)$ be the area of the shaded rectangle shown in the figure to the right. Show that $A(w)$ has its maximum value when w is the x-coordinate of the point of inflection of the graph of h.



(a) $$\text{volume} = 2\pi \int_0^\infty x e^{-x^2}\,dx$$

$$= 2\pi \lim_{b\to\infty} \int_0^b x e^{-x^2}\,dx = 2\pi \lim_{b\to\infty} -\frac{1}{2}e^{-x^2}\Big|_0^b$$

$$= 2\pi \lim_{b\to\infty} \left(-\frac{1}{2}e^{-b^2} + \frac{1}{2}e^0\right)$$

$$= 2\pi\left(\frac{1}{2}\right) = \pi, \quad \text{or}$$

$$\text{volume} = \pi \int_0^1 \left(\sqrt{(-\ln y)}\right)^2 dy$$

$$= -\pi \lim_{a\to 0+} \int_a^1 (\ln y)\,dy = \pi$$

5 $\begin{cases} 2: \text{writes } V \text{ as integral} \\ \quad 1: \text{integrand (includes constant)} \\ \quad 1: \text{limits} \\ 2: \text{expresses improper integral in terms of a proper integral} \\ 1: \text{answer from an improper integral} \end{cases}$

max 4/5: misuses infinity, 2 - 1 - 1.

(b) maximum:

$A(w) = we^{-w^2}$,

$A'(w) = e^{-w^2} - 2w^2e^{-w^2} = e^{-w^2}(1 - 2w^2)$.

$A'(w) > 0$ when $w < 1/\sqrt{2}$,

$A'(w) = 0$ when $w = 1/\sqrt{2}$,

$A'(w) < 0$ when $w > 1/\sqrt{2}$.

Therefore, max occurs when $w = 1/\sqrt{2}$.

inflection:

$h(x) = e^{-x^2}$, $h'(x) = -2xe^{-x^2}$,

$h''(x) = -2e^{-x^2} - 2x(-2x)e^{-x^2}$

$= 2e^{-x^2}(-1 + 2x^2)$.

$h''(x) < 0$ when $x < 1/\sqrt{2}$,

$h''(x) = 0$ when $x = 1/\sqrt{2}$,

$h''(x) > 0$ when $x > 1/\sqrt{2}$.

Therefore, inflection point when $x = 1/\sqrt{2}$.

Therefore, the maximum value of $A(w)$ and the inflection point of $h(x)$ occur when x and w are $1/\sqrt{2}$.

4 $\begin{cases} 1: A(w) \\ 1: \text{sets } A'(w) = 0 \\ 1: \text{sets } h''(x) = 0 \\ 1: \text{indicates that the equations have the same solution} \end{cases}$

QUESTION #2 / STANDARDS

BC-2

2. The Maclaurin series for $f(x)$ is given by $1+\frac{x}{2!}+\frac{x^2}{3!}+\frac{x^3}{4!}+\cdots+\frac{x^n}{(n+1)!}+\cdots$

(a) Find $f'(0)$ and $f^{(17)}(0)$.

(b) For what values of x does the given series converge? Show your reasoning.

(c) Let $g(x)=xf(x)$. Write the Maclaurin series for $g(x)$, showing the first three nonzero terms and the general term.

(d) Write $g(x)$ in terms of a familiar function without using series. Then, write $f(x)$ in terms of the same familiar function.

(a) $a_n=\dfrac{f^{(n)}(0)}{n!}=\dfrac{1}{(n+1)!}$

$$f'(0)=a_1=\frac{1}{2}$$

$$f^{(17)}(0)=17!\,a_{17}=17!\left(\frac{1}{18!}\right)=\frac{1}{18}$$

$$2\begin{cases}1: f'(0)\\ 1: f^{(17)}(0)\end{cases}$$

(b) $$\lim_{n\to\infty}\frac{\left|\dfrac{x^{n+1}}{(n+2)!}\right|}{\left|\dfrac{x^n}{(n+1)!}\right|}=\lim_{n\to\infty}\frac{|x|}{n+2}=0<1$$

converges for all x, by ratio test

$$2\begin{cases}1: \text{sets up ratio test and finds the limit}\\ 1: \text{applies ratio test to get answer}\end{cases}$$

(c) $g(x)=xf(x)$

$$=x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^{n+1}}{(n+1)!}+\cdots$$

$$2\begin{cases}1: \text{first three terms}\\ 1: \text{general term}\\ <-1>: \text{not an infinite series}\end{cases}$$

(d) $e^x=1+x+\dfrac{x^2}{2!}+\cdots+\dfrac{x^n}{n!}+\cdots$

$$e^x-1=g(x)=xf(x)$$

$$f(x)=\begin{cases}\dfrac{e^x-1}{x} & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$$

$$3\begin{cases}1: g(x)=e^x-1\\ 1: f(x)=\dfrac{e^x-1}{x}\\ \quad\text{(if } g(x) \text{ is incorrect must include } e^x\text{)}\\ 1: \text{correctly considers } x=0 \text{ case}\end{cases}$$

QUESTION #6 / STANDARDS

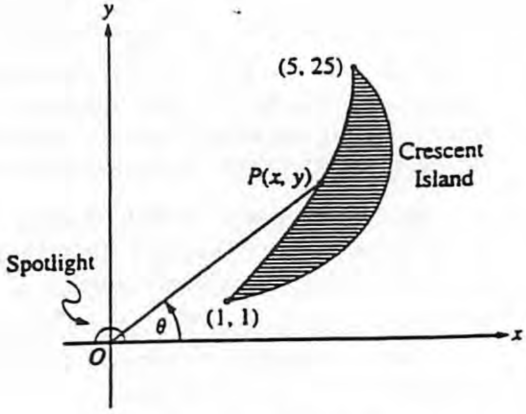
BC-6

6. The figure above shows a spotlight shining on point $P(x, y)$ on the shoreline of Crescent Island. The spotlight is located at the origin and is rotating. The portion of the shoreline on which the spotlight shines is in the shape of the parabola $y = x^2$ from the point $(1, 1)$ to the point $(5, 25)$. Let θ be the angle between the beam of light and the positive x-axis.

   (a) For what values of θ between 0 and 2π does the spotlight shine on the shoreline?

   (b) Find the x- and y-coordinates of point P in terms of $\tan\theta$.

   (c) If the spotlight is rotating at the rate of one revolution per minute, how fast is the point P traveling along the shoreline at the instant it is at the point $(3, 9)$?



Note: Figure not drawn to scale.

(a) $\tan\theta_1 = \dfrac{1}{1} \implies \theta_1 = \dfrac{\pi}{4}$ or 0.785

$\tan\theta_2 = \dfrac{25}{5} \implies \theta_2 = \tan^{-1} 5$ or 1.373

Therefore, $\dfrac{\pi}{4} \le \theta \le \tan^{-1} 5$

$$2\begin{cases} 1: \theta_1 \\ 1: \theta_2 \end{cases}$$

<−1> : no interval given

1/2 : complete solution in degrees

(b) $\tan\theta = \dfrac{y}{x} = \dfrac{x^2}{x} = x$

Therefore, $x = \tan\theta \qquad y = x^2 = \tan^2\theta$

$$2\begin{cases} 1: x = \tan\theta \\ 1: y = \tan^2\theta \end{cases}$$

(c) $\dfrac{d\theta}{dt} = 2\pi$

$$\frac{dx}{dt} = \sec^2\theta\,\frac{d\theta}{dt}; \quad \frac{dy}{dt} = 2\tan\theta\sec^2\theta\,\frac{d\theta}{dt}$$

at $(3, 9)$: $\dfrac{dx}{dt} = 10 \cdot 2\pi = 20\pi$

$$\frac{dy}{dt} = 2 \cdot 3 \cdot 10 \cdot 2\pi = 120\pi$$

speed $= \sqrt{(20\pi)^2 + (120\pi)^2}$

$= 20\pi\sqrt{37}$ or 382.191

$$5\begin{cases} 1: \dfrac{d\theta}{dt} = 2\pi \\ 1: \dfrac{dx}{dt} \text{ using Chain Rule} \\ 1: \dfrac{dy}{dt} \text{ using Chain Rule} \\ 1: \text{student uses magnitude of velocity} \\ 1: \text{answer} \end{cases}$$

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The following charts indicate the number of AP candidates from your school/district in each fifth of the total candidate group for each category described.

MULTIPLE-CHOICE

		LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN
DIFFERENTIAL CALCULUS	(25 Q.)			1	5	9	25	9.0	15.3
INTEGRAL CALCULUS	(15 Q.)			4	4	7	15	6.7	10.0
PART A- NO CALCULATOR	(25 Q.)			2	5	8	25	10.2	16.6
PART B- WITH CALCULATOR	(15 Q.)		1	3	4	7	15	5.5	8.8
TOTAL (40 QUESTIONS)				3	4	8	40	15.6	25.4
PERCENT IN EACH FIFTH				20%	27%	53%			
		LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN

AP GRADES	GRADE DISTRIBUTIONS	
	GLOBAL	GROUP
5	11.8%	40.0%
4	19.0%	26.7%
3	28.1%	33.3%
2	18.4%	0.0%
1	22.6%	0.0%
TOTAL CANDIDATES	102831	15

FREE-RESPONSE

	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN
QUESTION 1	1	2	3	4	5	9	3.5	4.9
QUESTION 2			2	1	12	9	4.8	8.2
QUESTION 3		1	2	4	8	9	5.0	7.1
QUESTION 4	3	3	3	3	3	9	4.4	4.5
QUESTION 5	1	1	3	1	9	9	2.4	5.3
QUESTION 6			2	4	9	9	2.2	4.7
TOTAL (FREE RESPONSE)		1	3	1	10	54	22.4	34.7
PERCENT IN EACH FIFTH		7%	20%	7%	67%			
	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH	MAX SCORE	GLOBAL MEAN	GROUP MEAN

9/95 -
Jim,
Re: AP
Results!
Jean

AI053595
VACAVILLE HIGH SCHOOL
AP COORD/JEAN BROOKS
100 MONTE VISTA AVENUE
VACAVILLE CA 95688

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FORM 3RBP

1995

The College Board
Advanced Placement Examination
CALCULUS AB
SECTION II

This green insert may be used for reference and/or scratchwork as you answer the free-response questions, but be sure to show all your work and your answers in the pink booklet. No credit will be given for work shown on this green insert.

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CALCULUS AB

SECTION II

Time — 1 hour and 30 minutes
Number of problems — 6
Percent of total grade — 50

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

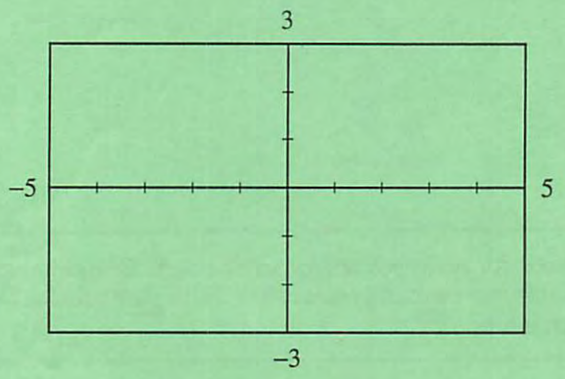
General instructions for this section are printed on the back cover of the test booklet.

---

1. Let f be the function given by $f(x) = \dfrac{2x}{\sqrt{x^2 + x + 1}}$.

(a) Find the domain of f. Justify your answer.

(b) In the viewing window provided below, sketch the graph of f.



Viewing Window
$[-5, 5] \times [-3, 3]$

(c) Write an equation for each horizontal asymptote of the graph of f.

(d) Find the range of f. Use $f'(x)$ to justify your answer.

Note: $f'(x) = \dfrac{x + 2}{(x^2 + x + 1)^{\frac{3}{2}}}$

GO ON TO THE NEXT PAGE

2. A particle moves along the y-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = t \cos t$. At time $t = 0$, the position of the particle is $y = 3$.

   (a) For what values of t, $0 \leq t \leq 5$, is the particle moving upward?

   (b) Write an expression for the acceleration of the particle in terms of t.

   (c) Write an expression for the position $y(t)$ of the particle.

   (d) For $t > 0$, find the position of the particle the first time the velocity of the particle is zero.

---

3. Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$.

   (a) Find $\frac{dy}{dx}$.

   (b) Write an equation for the line tangent to the curve at the point $(4, -1)$.

   (c) There is a number k so that the point $(4.2, k)$ is on the curve. Using the tangent line found in part (b), approximate the value of k.

   (d) Write an equation that can be solved to find the actual value of k so that the point $(4.2, k)$ is on the curve.

   (e) Solve the equation found in part (d) for the value of k.

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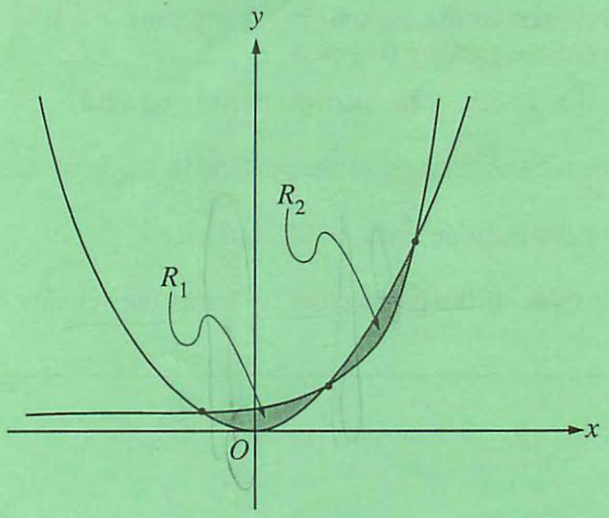

Note: Figure not drawn to scale.

4. The shaded regions R_1 and R_2 shown above are enclosed by the graphs of $f(x) = x^2$ and $g(x) = 2^x$.

(a) Find the x- and y-coordinates of the three points of intersection of the graphs of f and g.

(b) Without using absolute value, set up an expression involving one or more integrals that gives the total area enclosed by the graphs of f and g. Do not evaluate.

(c) Without using absolute value, set up an expression involving one or more integrals that gives the volume of the solid generated by revolving the region R_1 about the line $y = 5$. Do not evaluate.

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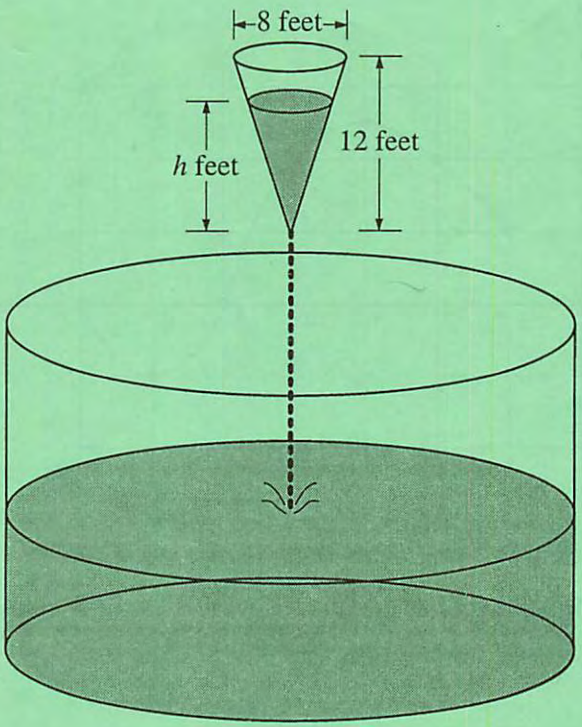

5. As shown in the figure above, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h - 12)$ feet per minute. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

   (a) Write an expression for the volume of water in the conical tank as a function of h.

   (b) At what rate is the volume of water in the conical tank changing when $h = 3$? Indicate units of measure.

   (c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure.

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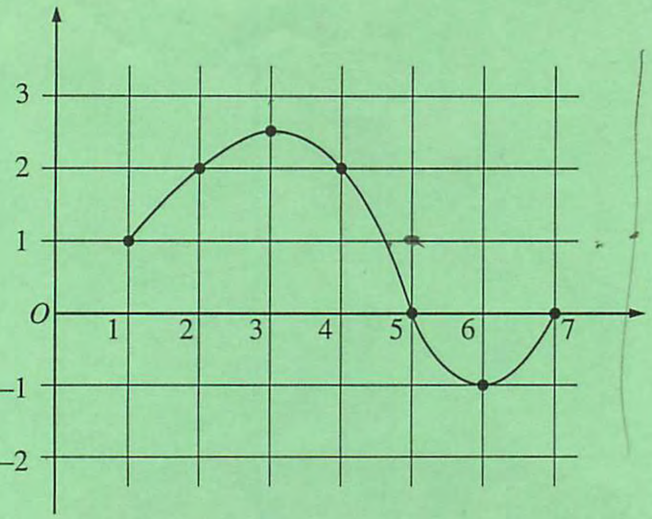


6. The graph of a differentiable function f on the closed interval $[1, 7]$ is shown above.

Let $h(x) = \int_1^x f(t)\,dt$ for $1 \le x \le 7$.

(a) Find $h(1)$.

(b) Find $h'(4)$.

(c) On what interval or intervals is the graph of h concave upward? Justify your answer.

(d) Find the value of x at which h has its minimum on the closed interval $[1, 7]$. Justify your answer.

END OF EXAMINATION

1995

① d) Range: $-\frac{4}{\sqrt{3}} \le f(x) < 2$

-2.039

② d) $\frac{\pi}{2}+2$

3.571

③ b) $y = 3x - 13$

④ b) $\int_{-.767}^{2} 2^x - x^2 \, dx + \int_{2}^{4} x^2 - 2^x \, dx$

28.274

⑤ b) "The volume is decreasing at 9π cu ft/min."

⑥ c) concave up for $1 \le x < 3$ And $6 < x \le 7$

Don't insist on endpts.

95

① $f(x) = \dfrac{2x}{\sqrt{x^2+x+1}}$

a) $x^2+x+1 \overset{\text{set}}{=} 0$

3pts

$x = \dfrac{-1 \pm \sqrt{1-4}}{}$

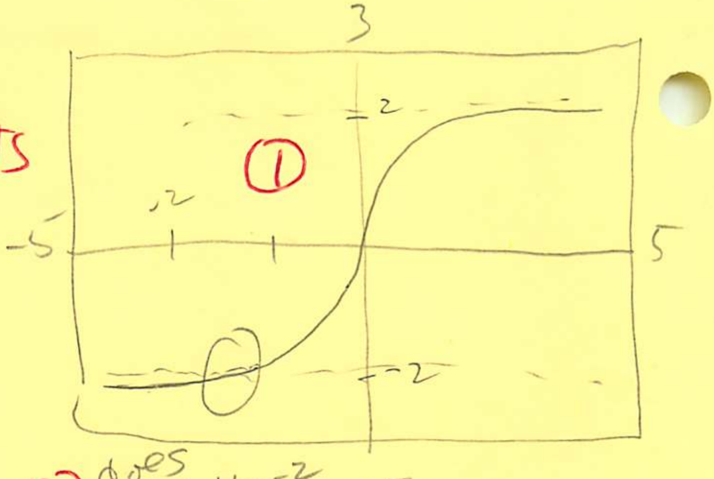
$x^2+x+1 > 0 \ \forall x$ ①

since $b^2 - 4Ac < 0$

$\therefore D_f \ x \in \mathbb{R}$ ②

b)

2pts



① does cross $y = -2$

c) $\displaystyle\lim_{x \to \infty} \frac{2x}{\sqrt{x^2+x+1}}$

4pts

$\displaystyle\lim_{x \to +\infty} \frac{2x}{x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}$ ①

$\displaystyle\lim_{x \to \infty} \frac{2}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}} \to 2 \ \therefore y = 2$

$\displaystyle\lim_{x \to -\infty} \frac{2x}{-x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}} \to -2$ ② $\therefore y = -2$

because $\sqrt{x^2} = |x|$.

The Horizontal tangents ① are $y = 2$ and $y = -2$

$\sqrt{4-2+1}$

D) $f'(x) = \dfrac{x+2}{(x^2+x+1)^{\frac{3}{2}}}$

6pts

$+/-$ f' $\begin{array}{ccc} - & 0 & + \\ \hline & -2 & \end{array}$ ①

$+$

① min at $x = -2$

$\left(-2, \frac{-4}{\sqrt{3}}\right)$ f' test.

$(-2, -2.309)$ ①

Range: $-\frac{4}{\sqrt{3}} \leq f(x) < 2$

↑③

Never gets to +2

→ Note f crosses its asymptote of $y = -2$ when $x = -1$

②

moves up and down 95

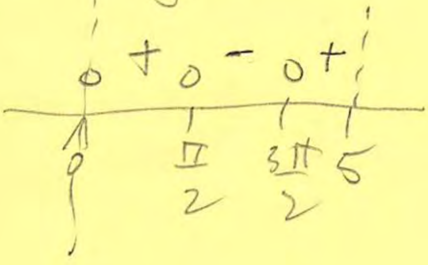
$v(t) = t\cos t$ $y(0) = 3$

a) moving up when ① $v(t) > 0$ $t\cos t \overset{\text{set}}{=} 0$ ①

4pts

$t = 0$, $\cos t = 0$

$\frac{\pi}{2}, \frac{3\pi}{2},$



$v(\frac{\pi}{4}) > 0$

1.571, 4.712

Particle moves up ~~down~~ for $0 < t < \frac{\pi}{2}$ and ② $\frac{3\pi}{2} < t \leq 5$

← include = because since moves up at end of time

b) acceleration = d(vel) ①

3pts

$a(t) = (\overset{u}{t}\ \overset{v}{\cos t})'$

$a(t) = \cos t - t\sin t$ ②

D) 1st time vel zero is π ①pT (for $t > 0$) See this line in part a.

$Y(\frac{\pi}{2}) = \frac{\pi}{2} \cdot 1 + 0 + 2$

$= \frac{\pi}{2} + 2$

c) position is $\int$ vel ①

7pts

$y(t) = \int t\cos t\,dt$ "parts" ←

$u = t$ $dv = \cos t\,dt$ ←

$du = dt$ $v = \sin t$ ③

↙

$y(t) = t\sin t - \int \sin t\,dt$ $\therefore y(t) = t\sin t + \cos t + 2$ ①

$t\sin t + \cos t + C$ ①

$y(0) = 0 \cdot 0 + 1 + c \overset{\text{set}}{=} 3$ ①

$\therefore c = 2$

95

③ $-8x^2 + 5\overset{u}{x}\overset{v}{y} + y^3 = -149$

) $-16x + 5y + 5x\overset{①}{\frac{dy}{dx}} + 3y^2\overset{①}{\frac{dy}{dx}} = 0$

3pts

$$\frac{dy}{dx} = \frac{16x - 5y}{5x+3y^2} \text{ ①}$$

b) $\left.\frac{dy}{dx}\right|_{4,-1} = \frac{16\cdot 4 + 5}{20 + 3} = \frac{69}{23} = 3 = m$ ②

4pts

$(4,-1)$ $m=3$

$y+1 = 3(x-4)$

$y = 3x - 13$ ②

c) $f(x+\Delta x) \approx f(x) + f'(x)\Delta x \approx K$

5pts

$x = 4$

$\Delta x = .2$

$f(4) + f'(4)(.2)$

$-1 + 3(.2)$

this method 4/5

$\approx -1 + .6$

$\approx -.4 = K$

or

$y = 3(4.2) - 13$

$y = 12.6 - 13$

$= -.4$ ⑤pts

so $K = -.4$

D) $-8(4.2)^2 + 5(4.2)y + y^3 = -149$

1pt

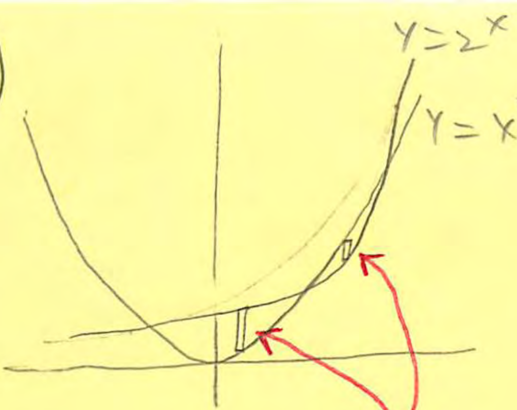
-141.12

$K^3 + 21K + 7.88 = 0$

E) $K = -.373$

2pts Solve $(x^3 + \underset{①}{21}x + 7.88, x, 1) = \underset{①}{-.373}$

④



1pT for showing cross section

a) 5pTS Solve $(x^2-2^x, x, -1) = -.767$

95

1) = 2

20) = 4

1 ea Answer OR 2 Justify

$f(x) = x^2-2^x$

Roots $-7.67, 2, 4$

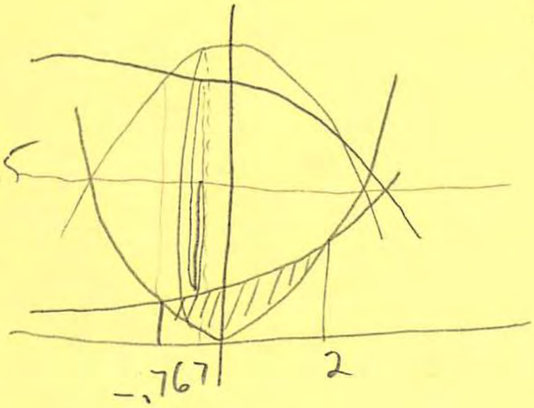
$(-7.67, .588)$

$(2, 4)$

$(4, 16)$

b) 5pTS $\int_{-.767}^{2} 2^x - x^2\,dx + \int_{2}^{4} x^2-2^x\,dx$ ④pT

c) pTS



"washers" ① $\pi\int r_1^2 - r_2^2\,dx$

$2^x \overset{\text{set}}{=} x^2 \Rightarrow -.767, 2$ from Part a.

outer radius   inner radius

$$\pi\int_{-.767}^{2} (5-x^2)^2 - (5-2^x)^2\,dx$$ ②

2pTS recogning radius of rev is $5-r_i$

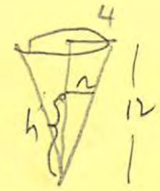
ⓑ If Done in y ....

$$\int_0^{.588} 2\sqrt{y}\,dy + \int_{.588}^{4} \sqrt{y} - \frac{\ln y}{\ln 2}\,dy + \int_4^{16} \frac{\ln y}{\ln 2} - \sqrt{y}\,dy$$

ⓒ If Done in y

$$2\pi\int_0^{.588} (5-y)\,2\sqrt{y}\,dy + 2\pi\int_{.588}^{4} (5-y)\left(\sqrt{y} - \frac{\ln y}{\ln 2}\right)dy$$

⑤ 4pts ①



$\frac{h}{r} = \frac{12}{4}$

$\frac{1}{3}h = r$ ①

a) $V = \frac{1}{3}\pi r^2 h$

$V_{cone} = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 \cdot h$

$V = \frac{\pi}{27}h^3$ ②

95

b) 4pts $\frac{dh}{dt} = h - 12$ note: $\frac{dh}{dt} < 0$ since $h < 12$

① $\left.\frac{dV}{dt}\right|_{cone} = ?$

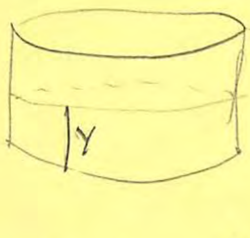
$V = \frac{\pi}{27}h^3$ ②

$\frac{dV}{dt} = \frac{\pi}{9}h^2\frac{dh}{dt}$

$= \frac{\pi}{9}h^2(h-12)\Big|_{h=3} = \frac{\pi}{9}\cdot 9\,(-9) = -9\pi$

① The vol. in cone is decreasing at 9π cubic feet per minute.

c) 7pts



$\left.\frac{dY}{dt}\right|_{h=3} = ?$

$V_{TANK} = \pi r^2 Y$ ① $\pi r^2 \overset{set}{=} 400\pi$, $r^2 = 400$ ①

$= 400\pi Y$

$\left.\frac{dV}{dt}\right|_{tank} = 400\pi\frac{dY}{dt}$ ①

$+9\pi = 400\pi\frac{dY}{dt}$

$\frac{9}{400} = \frac{dY}{dt}$ ①

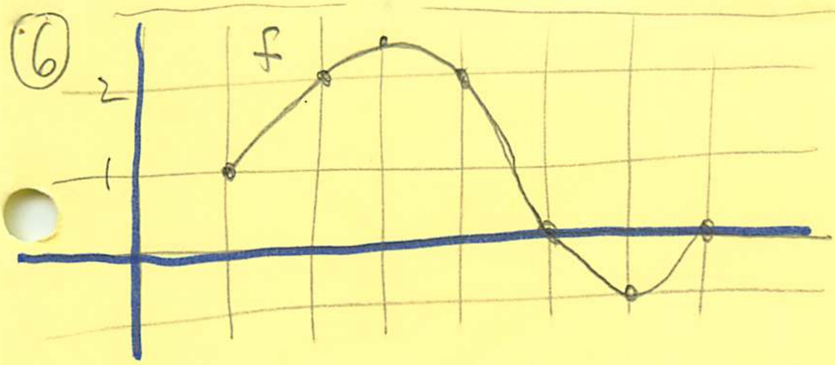
The Level of the water ① in the tank is rising at $\frac{9}{400}$ feet per minute.

② $\frac{dV}{dt}_{cone} = -\frac{dV}{dt}_{TANK}$ Since cone empties and tank fills.

note: volume is changing at constant rate; Rate of change of height of water is also constant.

ie no "h" in $\frac{dY}{dt}$.

⑥



1995

$h(x) = \int_1^x f(t)\,dt$

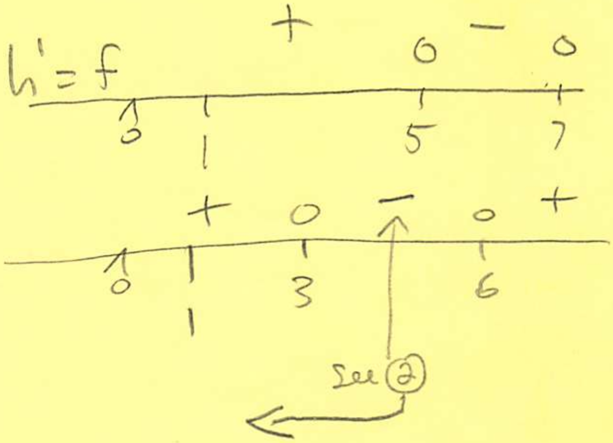
$1 \le x \le 7$

95

a) $h(1) = \int_1^1 f(t)\,dt = 0$ 2 pts

3 PTS b) $\underbrace{h'(4) \overset{①}{=} f(4)}_{2^{nd}\ fTC} = \overset{①}{2}$ ← Reading from graph ①

~~2 PTS~~ ~~1 Reasoning~~.



7 PTS c) h concave up ① when $h'' > 0$

① h'' has zeros when h' has extreme value 1 PT

h'' is 1st deriv of h' ie critical points

② PTS ② $h(5)$ is a max, f' test.

Since $x = 5$ is a max then $h''(5) < 0$ by f'' test. now other intervals alternate

h concave up $1 \le x < 3$ or $6 < x \le 7$ 3 PTS including inequalities

D) min occurs at $x = 1$ h increases from the END PT"

3 pts ALSO min occurs at $x = 7$ h decreases to the END pt"

1 each Answer

1 Justify

1c Alternative version: "$h'' > 0$ when h' is increasing. h' is $f(x)$ so. $h'' > 0$ when $f(x)$ has positive tangent il

$1 \le x < 3$ or $6 < x \le 7$

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1994 **ADMINISTRATION**

REPORT TO A[illegible]TEACHERS
F[illegible]

AI053595 VACAVILLE HIGH SCHOOL

MATHEMATICS - CALCULUS AB

The following charts indicate the number of AP candidates from your school/district in each fifth of the total candidate group for each category described.

MULTIPLE-CHOICE	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH
DIFFERENTIAL CALCULUS (28 Q.)	1		3	5	10*
INTEGRAL CALCULUS (16 Q.)	1	1	6	5*	6
TOTAL (44 QUESTIONS)	1		5	3	10*
** PERCENT IN EACH FIFTH	5%		26%	16%	53%*

TOTAL AP GROUPS BY FIFTHS

FREE-RESPONSE	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH
QUESTION 1	1	1		7	10*
QUESTION 2	2		5	4*	8
QUESTION 3	3		2	8*	6
QUESTION 4		3	4	4*	8
QUESTION 5			2	8*	9
QUESTION 6	1		3	5	10*
TOTAL (FREE RESPONSE)	1	1	2	4	11*
** PERCENT IN EACH FIFTH	5%	5%	11%	21%	58%*

TOTAL AP GROUPS BY FIFTHS

AI053595
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VACAVILLE CA 95688

* = Location of median (50th percentile) for school/district if given category includes 5 or more candidates

** = Percentage of your students in each fifth for the entire section of the exam

Total number of AP candidates in subject for year = 93323

Number of candidates in your group = 19

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1994 CALCULUS AB

Show all your work. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your final answers. If you choose to use decimal approximations, your answer should be correct to three places after the decimal point.

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

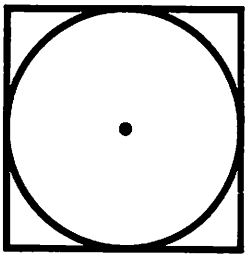
1. Let f be the function given by $f(x) = 3x^4 + x^3 - 21x^2$
   - a) Write an equation of the line tangent to the graph of f at the point $(2, -28)$.
   - b) Find the absolute minimum value of f. Show the analysis that leads to your conclusions.
   - c) Find the x – coordinate of each point of inflection on the graph of f. Show the analysis that leads to your conclusion.

2. Let R be the region enclosed by the graphs of $y = e^x$, $y = x$, and the lines $x = 0$ and $x = 4$.
   - a) Find the area of R.
   - b) Find the volume of the solid generated when R is revolved about the x – axis.
   - c) Set up , but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y – axis.

3. Consider the curve defined by $x^2 + xy + y^2 = 27$.
   - a) Write an expression for the slope of the curve at any point (x, y).
   - b) Determine whether the lines tangent to the curve at the x – intercepts of the curve are parallel. Show the analysis that leads to your conclusion.
   - c) Find the points on the curve where the lines tangent to the curve are vertical.

4. A particle moves along the x – axis so that at any time $t > 0$ its velocity is given by $v(t) = t\ln t - t$. At time $t = 1$, the position of the particle is $x(1) = 6$.
   - a) Write an expression for the acceleration of the particle.
   - b) For what values of t is the particle moving to the right?
   - c) What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.
   - d) Write an expression for the position $x(t)$ of the particle.

5. A circle is inscribed in a square as shown in the figure. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency.
   - a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.
   - b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.



6. Let $F(x) = \int_0^x \sin(t^2)\, dt$ for $0 \le x \le 3$.
   - a) Use the trapezoidal rule with four equal subdivisions of the closed interval [0, 1] to approximate $F(1)$.
   - b) On what intervals is F increasing?
   - c) If the average rate of change of F on the closed interval [1, 3] is k, find $\int_1^3 \sin(t^2)\, dt$ in terms of k.

1994

① c) $x = \{ \frac{-7}{6}, 1 \}$ must show f'' changes signs

② b) $\frac{\pi}{6}(3e^8 - 131) = 4613.886$

③ c) $(-6, 3)$ $(6, -3)$

④ d) $x(t) = \frac{1}{2}t^2 \ln t - \frac{3t^2}{4} + \frac{27}{4}$

⑤ b) $(4 - \pi)\frac{30}{\pi} = 8.197$ in²/sec

⑥ b) $0 < x < \sqrt{\pi}$ and $\sqrt{2\pi} < x < 3$

1,772 2,507

①

$f(x) = 3x^4 + x^3 - 21x^2$

$x^2(3x^2 + x - 21)$

organization ①

⑤ a) Tangent line need $m\big|_{x=2} = f'(2)$ and $(2, -28)$

$f'(x) = 12x^3 + 3x^2 - 42x$ ①

$3x(4x^2 + x - 14)$

$3x(4x - 7)(x + 2)$

$f'(2) = 6(8-7)(4)$

$= 24$ ①

$y - y_1 = m(x - x_1)$

$y + 28 = 24(x - 2)$

$= 24x - 48$

method ①

$y = 24x - 76$ ①

⑥ b) Abs min

①

$$f' \quad \begin{array}{ccccccc} - & 0 & + & 0 & - & 0 & + \\ & -2 & & 0 & & \frac{7}{4} & \quad 2 \end{array}$$

2 minimums verified ①

min at -2, $f(-2)$ and $\left(\frac{7}{4}, f\left(\frac{7}{4}\right)\right)$ by f' test.

$f(-2) = (-2)^2(3(4) - 2 - 21)$

$4(12 - 23)$

-44 ①

$f\left(\frac{7}{4}\right) = \frac{49}{16}\left(3 \cdot \frac{49}{16} + \frac{7}{4} - 21\right)$

$\frac{49}{16}(-10.0625)$

$= -30.816$ ①

② The Abs min of f is -44 (when $x = -2$) since it is the lowest of the only two mins.

④ c) PI when f'' changes signs.

①

$$f'' \quad \begin{array}{ccccc} + & 0 & - & 0 & + \\ & -\frac{7}{6} & 0 & 1 & \end{array}$$

$f''(x) = 36x^2 + 6x - 42$

① $6(6x^2 + x - 7)$

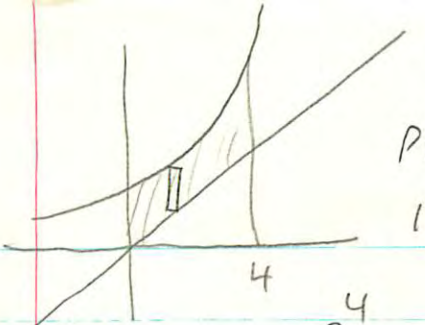
$6(6x + 7)(x - 1)$

PI occur when $x \in \left\{-\frac{7}{6}, 1\right\}$ ① because f changes concavity at those values. ①

1994

②

(alternatives on next page)



Pict ② including cross section

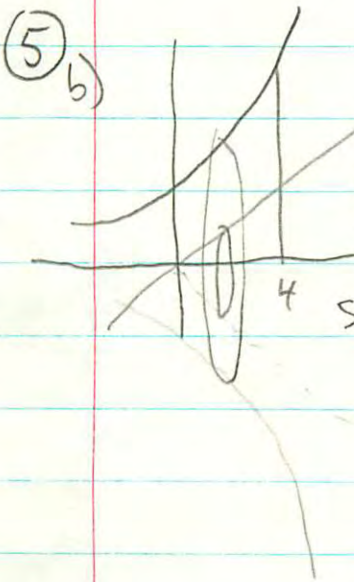
⑥ a) area $\int_0^4 e^x - x\,dx$ ②

$$e^x \overset{①}{-} \frac{1}{2}x^2\Big|_0^4 = e^4 - 8 - (1-0)$$

$$= e^4 - 9$$ is the area of R

① 45.598

⑤ b)



Pict of ① sample

"washers" or ① $\pi\int r_1^2 - r_2^2\,dx$

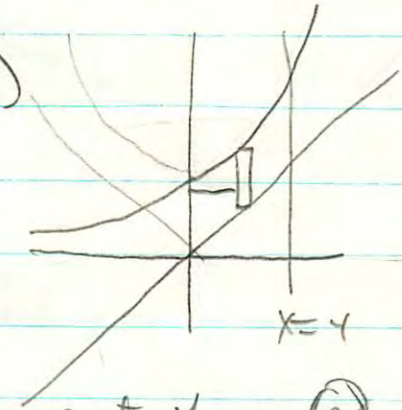
$$\pi\int_0^4 e^{2x} - x^2\,dx \quad ②$$

$$\pi\left(\frac{1}{2}e^{2x} - \frac{1}{3}x^3\right)\Big|_0^4$$

$$\pi\left(\frac{1}{2}e^8 - \overset{-128}{\frac{64}{3}} - \overset{-3}{\frac{1}{2}}\right) = \frac{\pi}{6}\left(3e^8 - 131\right) \quad ①$$

4613.886

④ c)



"shells" ① $2\pi\int$ radius · shell

$$2\pi\int_0^4 x\left(e^x - x\right)dx$$

①

Pict of sample including radius of rev. ②

IN "y"

$$V_y = \pi\left[\int_0^1 y^2\,dy + \int_1^4 y^2 - (\ln y)^2\,dy + \int_4^{e^4} 16 - (\ln y)^2\,dy\right]$$

alternatives. 1994 ②

a)

Using geometry (area of Triangle)

$$\int_0^4 e^x\,dx - \frac{1}{2}\cdot 4\cdot 4$$

3:
- 2: Integral
- 1: Antidifferentiation and evaluation

or Using y-axis

$$\int_0^1 y\,dy + \int_1^4 y-\ln y\ dy + \int_4^{e^4} 4-\ln y\ dy$$

or

3:
- 2: Integral
  - 1: Limits
  - 1: All 3 integrands correct
- Note: 0/2 if any limits incorrect
- 1: Answer

b)

Using geometry (Volume of cone)

$$\pi\int_0^4 (e^x)^2\,dx - \frac{1}{3}\pi\cdot 4^2\cdot 4$$

$$= \pi\left(\frac{1}{2}e^{2x}\right)\Big|_0^4 - \frac{\pi}{3}\cdot 64$$

$$= \pi\left(\frac{1}{2}e^8 - \frac{1}{2}\right) - \frac{64\pi}{3}$$

4:
- 3: Integral
  - 1: π and correct limits
  - 2: Correct
    - <-1> e^{x^2}
- 1: Antidifferentiation and evaluation
- 0/1 if student doesn't have correct cone volume or doesn indicate using cone.

Using y-axis

$$2\pi\left[\int_0^1 y\cdot y\,dy + \int_1^4 y(y-\ln y)\,dy + \int_4^{e^4} y(4-\ln y)\,dy\right]$$

4:
- 3: Integral
  - 1: 2π and limits
  - 2: integrands
    - <-1> each error
  - Note: 0/2 if any limits incorrect
- 1: Answer

1994
③

$x^2 + \overset{u}{x}\overset{v}{y} + y^2 = 27$

④ a) $\frac{dy}{dx} \overset{①}{=} ?$ $\quad 2x + \overset{①}{\left(y + x\frac{dy}{dx}\right)} + \overset{①}{\left(2y\frac{dy}{dx}\right)} = 0$

$$\frac{dy}{dx} = \frac{-(2x+y)}{x+2y} \quad ①$$

⑤ b) x inter of curve when $y=0$ $\Rightarrow x^2 + 0 + 0 = 27$ ESTAblish more than

$x = \pm 3\sqrt{3}$ ① 1 tangent

x inter are in the form $(x_i, 0)$

② $\left.\frac{dy}{dx}\right|_{(x_i,0)} = \frac{-2x_i}{x_i} = -2$ both Tangents have Slope -2 therefore both tangents are parallel ①

⑥ c) coord where Tang DNE. when $\frac{dy}{dx}$ ① DNE ... when denom $=0$

denom of $\frac{dy}{dx} = x + 2y \overset{\text{set}}{=} 0$ ① $\quad x^2 + xy + y^2 = 27$

$x = -2y \quad 4y^2 - 2y^2 + y^2 = 27$

$3y^2 = 27$

$x = \mp 6 \qquad y = \pm 3$ ①

∴ $(-6, 3)$ and $(6, -3)$ are

③ The two pts where the Tangent line is vertical.

1994
④

$v = \overset{u}{t}\,\overset{v}{\ln t} - t$

③ a) accel = ? $a = \overset{①}{d}(v)$ $a = \ln t + t\cdot\frac{1}{t} - 1$

$a = \ln t$ ②

④ b) moves right when $v > 0$ ①

$t\ln t - t \overset{\text{set}}{=} 0$ v $-\ 0\ +$ ①
$0 \qquad e$

$t(\ln t - 1) = 0$

$t = 0$ $\ln t = 1$ ①

Reject $t = e$ Particle moves right when $t > e$ ①

$t > 0$

④ c) min velocity $d(v) \overset{\text{set}}{=} 0$ ① $\ln t \overset{\text{set}}{=} 0$ a $-\ 0\ +$ ①
$t = 1$ $0 \qquad 1$

min occurs when $t = 1$ f' Test ①

That min velocity is $v(1) = 1\cdot 0 - 1 = -1$ ①

④ D) $x(t) = \int v\,dt$ ①

"parts" $\int t\ln t - \cancel{t}\,dt$

$u = \ln t$ $dv = t\,dt$

② $du = \frac{1}{t}dt$ $v = \frac{1}{2}t^2$

$\frac{1}{2}t^2\ln t - \frac{1}{2}\int t\,dt - \frac{t^2}{2}$

$\frac{1}{2}t^2\ln t - \frac{t^2}{4} - \frac{t^2}{2} + C$

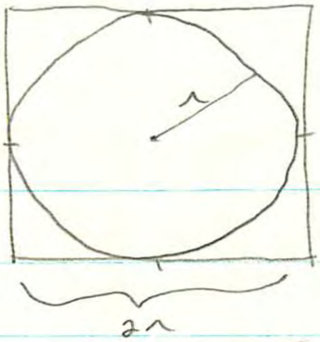
$x(1) = \frac{1}{2}\cdot 1\cdot 0 - \frac{1}{4} - \frac{1}{2} + c \overset{\text{set}}{=} 6$

$c = 6\frac{3}{4}$

$= \frac{27}{4}$

$x(t) = \frac{1}{2}t^2\ln t - \frac{3t^2}{4} + \frac{27}{4}$ ①

1994 ⑤



$\frac{dc}{dt} = 6$ ①

⑦ a) $\frac{dP}{dt} = ?$ ① $P = 8r$ ①

$c = 2\pi r$

$\frac{dc}{dt} = 2\pi \frac{dr}{dt}$

$6 = 2\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{3}{\pi}$ ②

$\frac{dP}{dt} = 8 \frac{dr}{dt}$

$= 8\left(\frac{3}{\pi}\right)$

$= \frac{24}{\pi}$ IN/sec. (linear)
① ①

The Perimeter is growing at $\frac{24}{\pi}$ IN/sec.

⑧ b) $\left.\frac{dA}{dt}\right| = ?$ ①

area $= 25\pi$

② $A = 4r^2 - \pi r^2$ area of shade

$\frac{dA}{dt} = 8r \frac{dr}{dt} - 2\pi r \frac{dr}{dt}$ ①

$\pi r^2 = 25\pi$

$r = 5$

①

$\frac{dA}{dt} = 40 \frac{3}{\pi} - 10\pi \cdot \frac{3}{\pi}$ ①

$\frac{dA}{dt} = (4-\pi)\frac{30}{\pi}$ IN²/sec. (area)
① ①

The difference in the areas is growing at the rate of

$(4-\pi)\frac{30}{\pi}$ IN²/sec

8.197 in²/sec.

1994
⑥

$f(x) = \int_0^x \sin t^2\,dt \qquad 0 \le x \le 3$

⑦ a) interval [0, 1] 4 sections each $\frac{1}{4}$ wide $\therefore h = \frac{1}{4}$ ①

$x_i =$ ① $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$ $\qquad \frac{h}{2}\left(f(x_0) + 2f(x_1) + \dots 2f(x_{n-1}) + f(x_n)\right]$ ①

$\frac{1}{8}\left[0 + 2(.0624) + 2(.2474) + 2(.5333) + .8415\right]$ ③ show work!

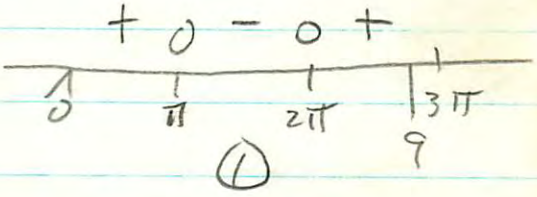
$.125 \qquad .4948 \qquad 1.0666$

$\frac{1}{8}\left[2.528\right] = .316$ ①

⑤ b) $f' = \sin x^2 \overset{\text{set}}{=} 0$ $\qquad$ f increases when $f' > 0$ ①

let $x^2 = a$ $\qquad x = 0 \quad a = 0$ $\qquad \sin a \overset{?}{>} 0$

$x = 3 \quad a = 9$



f increases for $0 < x < \sqrt{\pi}$ ③
and $\sqrt{2\pi} < x < 3$

③ c) ave val $= \frac{1}{b-a}\int_a^b f(x)\,dx$ ①

$= \frac{1}{3-1}\int_1^3 \sin t^2\,dt \overset{\text{set}}{=} K$

$\int_1^3 \sin t^2\,dt = 2K.$ ②

Calculus AB

Show all your work. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your final answers. If you choose to use decimal approximations, your answer should be correct to three decimal places.

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Let f be a function given by $f(x) = x^3 - 5x^2 + 3x + k$, where k is a constant.

   a) On what intervals is f increasing?
   b) On what intervals is the graph of f concave downward?
   c) Find the value of k for which f has 11 as its relative minimum.

2. A particle moves on the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = 2te^{-t}$.

   a) Find the acceleration of the particle at $t = 0$.
   b) Find the velocity of the particle when its acceleration is 0.
   c) Find the total distance traveled by the particle from $t = 0$ to $t = 5$.

3. Consider the curve $y^2 = 4 + x$ and the chord AB joining points $A(-4, 0)$ and $B(0, 2)$ on the curve.

   a) Find the x– and y–coordinates of the point on the curve where the tangent line is parallel to chord AB.
   b) Find the area of the region R enclosed by the curve and chord AB.
   c) Find the volume of the solid generated when the region R, defined in part (b), is revolved about the x–axis.

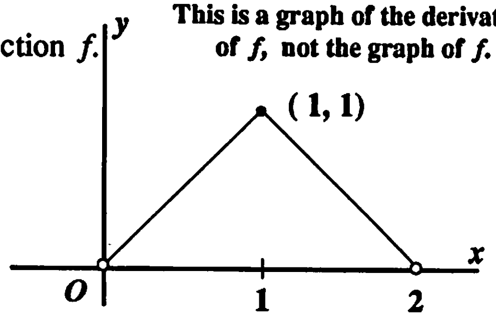
4. Let f be the function defined by $f(x) = \ln(2 + \sin x)$ for $\pi \leq x \leq 2\pi$.

   a) Find the absolute maximum value and the absolute minimum value of f. Show the analysis that leads to your conclusion.
   b) Find the x–coordinate of each inflection point on the graph of f. Justify your answer.

5. The figure to the right shows the graph of f', the derivative of the function f. The domain of f is the set of all x such that $0 < x < 2$.

   a) Write an expression for $f'(x)$ in terms of x.
   b) Given that $f(1) = 0$, write an expression for $f(x)$ in terms of x.
   c) Sketch the graph of $y = f(x)$.

This is a graph of the derivative of f, not the graph of f.



6. Let $P(t)$ represent the number of wolves in a population at time t years, when $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $800 - P(t)$, where the constant of proportionality is the positive constant k.

   a) If $P(0) = 500$, find $P(t)$ in terms of t and k.
   b) If $P(2) = 700$, find k.
   c) Find $\lim\limits_{t \to \infty} P(t)$.

1993

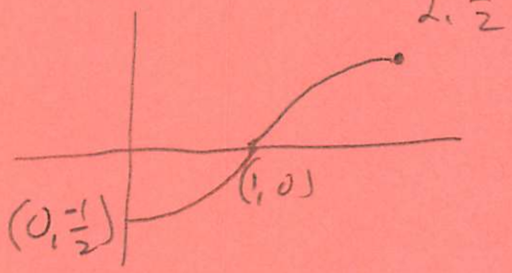
① c) $K = 20$

② c) $\frac{4}{e} - \frac{10}{e^5} = 1.404$

③ c) $\frac{8\pi}{3} = 8.378$

④ Abs min value zero
Abs max value $\ln 2$

⑤ c)



⑥ b) $K = -\frac{1}{2} \ln 3 = -.549$

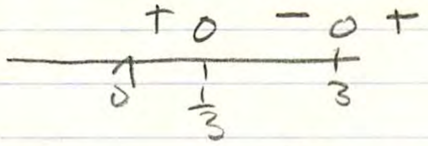
1993
①

$f(x) = x^3 - 5x^2 + 3x + k$

Der

⑤ a) f increasing? ① $f'(x) = 3x^2 - 10x + 3 \overset{set}{=} 0$

Justify ② f'



$(3x - 1)(x - 3)$

$x \in \{\frac{1}{3}, 3\}$

f increases for $x < \frac{1}{3}$ and for $x > 3$

f increases when $f' > 0$ ② Proper response

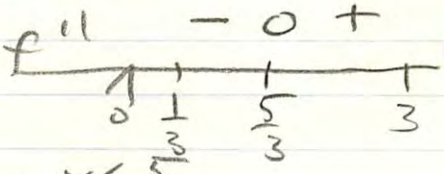
b)

⑤ b) f concave down when $f'' < 0$ ① Explain

$f''(x) = 6x - 10$ ① Der

$2(3x - 5)$

f''



② Justify

f concave down when $x < \frac{5}{3}$

①

⑤ c) f has min value of 11 ....

③ f has min value only when $x = 3$ by f' test

$f(3) = 27 - 45 + 9 + k \overset{set}{=} 11$ ①

$-9 + k =$

$k = 20$ ①

1993
②

$\tau \geq 0 \quad x(\tau) = \overset{u}{(2\tau)}\overset{v}{e^{-\tau}}$

④ a) acceleration is 2nd der of x ①

$v(\tau) = 2e^{-\tau} + 2\tau e^{-\tau}(-1)$

$= 2e^{-\tau} - 2\tau e^{-\tau}$

① $v = 2e^{-\tau}(1-\tau)$

$a(\tau) = -2e^{-\tau} - (2e^{-\tau} - 2\tau e^{-\tau})$

① $= -4e^{-\tau} + 2\tau e^{-\tau}$

$a(\tau) = -2e^{-\tau}(2-\tau)$

① $a(0) = -2(1)(2) = -4$

⑤ b) acc = 0 when $\overset{①}{\underset{\overset{\uparrow}{\text{① Never zero}}}{-2e^{-\tau}}}(2-\tau) = 0$

$\tau = 2$ only solution ①

① find $v(2) =$

$v(\tau) = 2e^{-\tau}(1-\tau)$ ✓

$v(2) = 2e^{-2}(1-2)$

$= -\frac{2}{e^2}$ ① $= -0.271$

⑥ c) v' ① $\underset{0}{\overset{+}{\longrightarrow}}\; \underset{1}{0}\; -$

Total Dist = $\overset{①}{\underbrace{\text{Dist}[0,1]}}$ + Dist [1,5]

$\left|\frac{2}{e} - 0\right| + \left|\frac{10}{e^5} - \frac{2}{e}\right|$

② $\begin{cases} x(0) = 0 \\ x(1) = \frac{2}{e} \\ x(5) = \frac{10}{e^5} \end{cases}$

how absVal removed

① since $\frac{10}{e^5} - \frac{2}{e}$ is Left movement

must use def of |A|

$\frac{2}{e} - \left(\frac{10}{e^5} - \frac{2}{e}\right)$

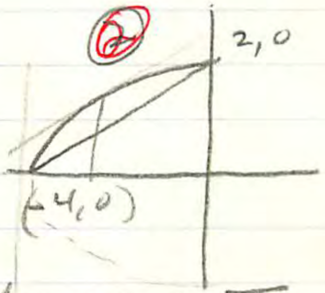
$\frac{2}{e} - \frac{10}{e^5} + \frac{2}{e} = \frac{4}{e} - \frac{10}{e^5}$ ①

1,47152 − .06733

1,404

1993
③

⑧ a) $y^2 = 4+x$ A$(-4,0)$ B$(0,2)$



$y = \pm\sqrt{4+x}$

① $m = \frac{2-0}{0+4} = \frac{1}{2}$ ∴ chord: $y = \frac{1}{2}x + 2$

① we are using top ½ of relation ∴ choose +√

$y = \sqrt{4+x}$

② $y' = \frac{1}{2\sqrt{4+x}} \overset{set}{=} \frac{1}{2}$

$\frac{1}{\sqrt{4+x}} = 1$

$\sqrt{4+x} = 1$

$4+x = 1$

$x = -3$

$y^2 = 4-3$

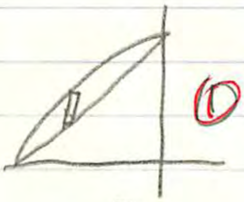
$y^2 = 1$

$y = \pm 1$ ①

① Proper chord

COORDINATES $(-3, 1)$

③ b)



①

① $\int_{-4}^{0} \sqrt{4+x} - (\frac{1}{2}x+2)\,dx$

$\frac{2}{3}(4+x)^{\frac{3}{2}} - \frac{1}{4}x^2 - 2x \Big|_{-4}^{0}$

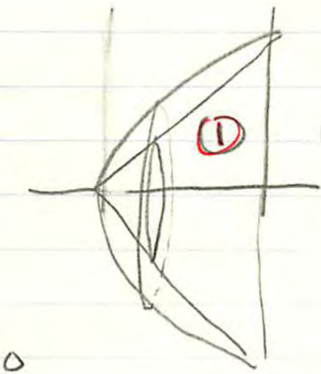
$\frac{2}{3}(4)^{\frac{3}{2}} - 0 - \left[0 - \frac{1}{\cancel{4}}\overset{4}{\cancel{16}} + 8\right]$

$\frac{2}{3} \cdot 8 - [4]$

$\frac{16}{3} - \frac{12}{3}$

① $\frac{4}{3}$ = 1.333

④ c)



① washers

$\pi\int_a^b r_1^2 - r_2^2\,dx$

① $\pi\int_{-4}^{0} (4+x) - (\frac{1}{2}x+2)^2\,dx$

$\pi\int_{-4}^{0} \cancel{4}+x - \frac{1}{4}x^2 - 2x - \cancel{4}\,dx$

$\pi\int_{-4}^{0} -\frac{1}{4}x^2 - x\,dx$

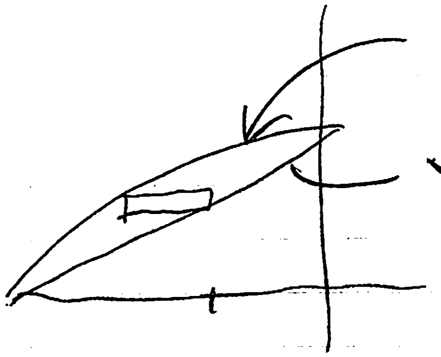
$\pi\left(-\frac{1}{12}x^3 - \frac{1}{2}x^2\right)\Big|_{-4}^{0}$

$\pi\left(0 - \left(-\frac{1}{\cancel{12}_{3}}(\overset{16}{\cancel{-64}}) - \frac{1}{\cancel{2}}\cdot\overset{8}{\cancel{16}}\right)\right)$ 1

$\pi\left(-\left(\frac{16}{3} - \frac{24}{3}\right)\right)$

washers volume → $\frac{8\pi}{3}$ ① = 8.378

Horizontal sections.



$y^2-4=x$

$y=\frac{1}{2}x+2 \therefore 2y-4=x$

$$\int_0^2 2y-4-(y^2-4)\,dy$$

$$\int_0^2 2y-4-y^2+4\,dy$$

$$\int_0^2 2y-y^2\,dy$$

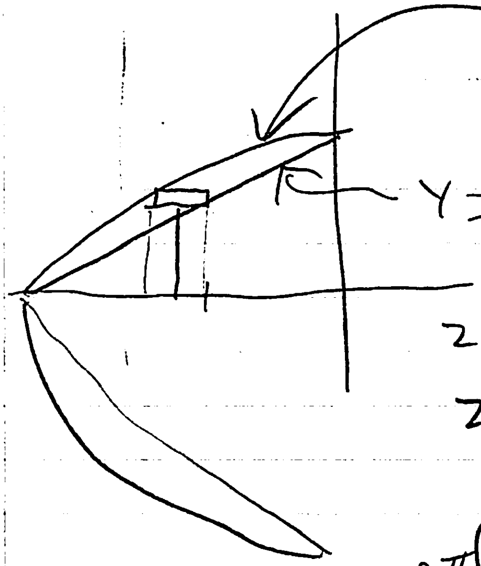
$$y^2-\frac{y^3}{3}\Big|_0^2 = 4-\frac{8}{3}$$

$$\frac{12-8}{3}=\frac{4}{3}$$

1993
3c

$y = +\sqrt{4+x}$

shells. $y^2 - 4 = x$



$y = \frac{1}{2}x + 2$

$2y = x + 4$

$2y - 4 = x$

$2\pi\int$ radius (length of cross Section)

$$2\pi\int_0^2 y\left[(2y-4)-(y^2-4)\,dy\right.$$

$$2\pi\int_0^2 y\{(2y-\cancel{4}-y^2+\cancel{4})]\,dy$$

$$-2\pi\int_0^2 y^3 - 2y^2\,dy$$

$$-2\pi\left(\frac{y^4}{4}-\frac{2y^3}{3}\right)\Big|_0^2$$

$$-2\pi\left(\frac{\overset{4}{\cancel{16}}}{\cancel{4}}-\frac{2\cdot 8}{3}\right)$$

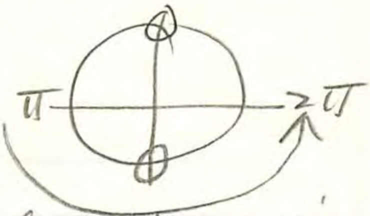
$$-2\pi\left(\frac{12-16}{3}\right)$$

$$-2\pi\left(\frac{-4}{3}\right)$$

$$\frac{8\pi}{3}$$

$f(x) = \ln(2+\sin x)$ $\quad \pi \le x \le 2\pi$ $\quad$ 1993 ④

⑨ a) $f'(x) = \dfrac{{}^{u}\ \cos x}{{}_{v}\ 2+\sin x} \overset{set}{=} 0$ ①



$\cos x = 0$

$x = \frac{3\pi}{2}$ only value in domain.

#line method to Determine ⑨

f' ② $\quad | - \ 0 \ + |$

$0 \quad \pi \quad \frac{3\pi}{2} \quad 2\pi$

① $2 + \sin x$ always > 0
$\cos x > 0$ $\frac{3\pi}{2} \to 2\pi$

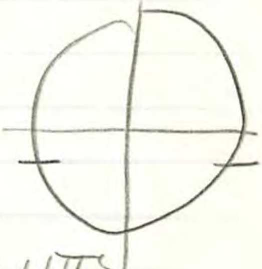
$f(\pi) = \ln(2+0) = \ln 2$
$f(\frac{3\pi}{2}) = \ln(2-1) = \ln 1 = 0$
$f(2\pi) = \ln(2+0) = \ln 2$

② $(\frac{3\pi}{2}, 0)$ abs min f' test
② $(\pi, \ln 2)$ abs max decrease fm it
② $(2\pi, \ln 2)$ abs max increase to it
includes Justifications

⑥ B) $f''(x) = \dfrac{(2+\sin x)(-\sin x) - \cos x(\cos x)}{(2+\sin x)^2}$

$$\frac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2}$$

$$\frac{-2\sin x - (\sin^2 x + \cos^2 x)}{(2+\sin x)^2}$$



② $f'' = \dfrac{-(1+2\sin x)}{(2+\sin x)^2}$ $\quad \sin x = -\frac{1}{2}$

$x = \{\frac{7\pi}{6}, \frac{11\pi}{6}\}$

② $f''(0) = \dfrac{(-)(+)}{+} = (-)$
including #line

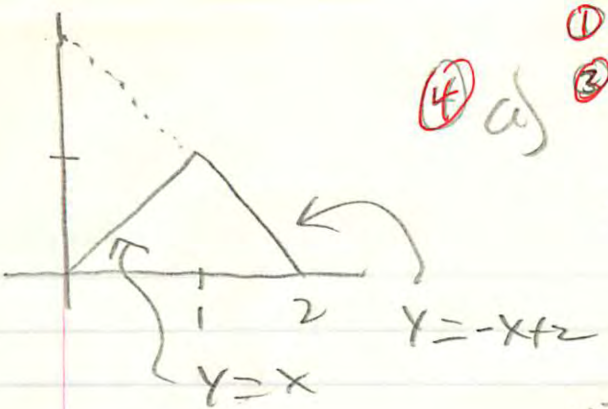
f'' $\quad - \ 0 \ + \ 0 \ -$

$0 \quad \pi \quad \frac{7\pi}{6} \quad \frac{11\pi}{6} \quad 2\pi$

Since f changes from concave down to
② concave up at $\frac{7\pi}{6}$ and from up to down at $\frac{11\pi}{6}$ These are the x values of the PI,

① Recognize Piecemeal fctn
② Proper ~~[illegible]~~ Domains. 1993

④ a) ⑤



$$f'(x)=\begin{cases} x & \text{if } 0<x<1 \\ -x+2 & \text{if } 1\le x<2 \end{cases}$$

Note: f' could be defined at x=1 in either the Top or bottom interval.

⑧ ⓑ $f(1)=0$

Note f' exists for x in (0, 2) including x=1. ∴ f must also be continuous.

② Justify & method of det constant.

$$\int f'\,dx = f := \begin{cases} \frac{x^2}{2}+c_1 & \text{if } 0<x<1 \\ -\frac{x^2}{2}+2x+c_2 & \text{if } 1\le x<2 \end{cases}$$

② preliminary results including ~~constants~~ of integration

$$\lim_{x\to 1^-} f(x) \overset{\text{must}}{=} \lim_{x\to 1^+} f(x) = 0$$

$\frac{1}{2}^2+c_1=0$ $\quad$ $\frac{-1}{2}+2+c_2=0$

$c_1=\frac{-1}{2}$ $\quad$ $\frac{3}{2}+c_2=1$

③ $c_2=-\frac{3}{2}$

Actual Values of Constant.

final answer

① $$\therefore f(x)=\begin{cases} \frac{x^2}{2}-\frac{1}{2} & 0<x<1 \\ -\frac{x^2}{2}+2x-\frac{3}{2} & 1\le x<2 \end{cases}$$

③

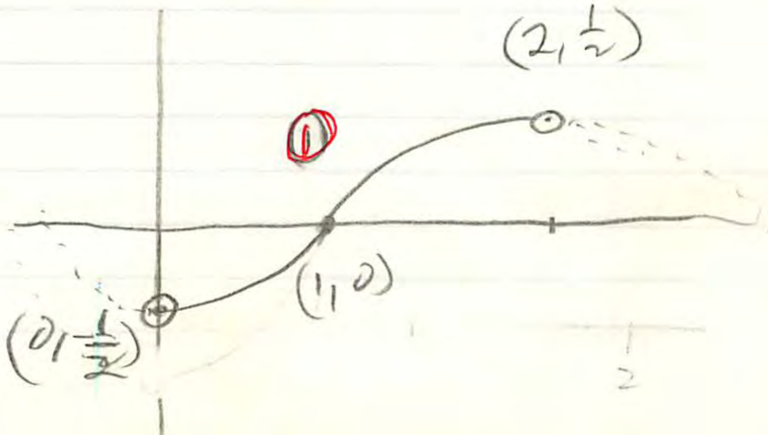
c) $y=\frac{1}{2}x^2-\frac{1}{2}$ Parabola vertex $(0,-\frac{1}{2})$ opens up. ①

$y=\frac{-1}{2}x^2+2x-\frac{3}{2}$

$y=-\frac{1}{2}(x^2-4x+4)+\frac{1}{2}$

① $-\frac{1}{2}(x-2)^2+\frac{1}{2}$

Parabola vertex $2,\frac{1}{2}$ opens down



6

1993

"The pop INC at a rate directly prop"

⑨ A) ① $\frac{dP}{dt} = K(800-P)$

Turned around $\frac{dP}{P-800} = -K\,dt$

② $\int \frac{dP}{P-800} = -\int K\,dt$

$\ln(P-800) = -Kt + C$ ①

$P - 800 = e^{-Kt+C}$

$P = 800 + e^{-Kt+C}$

① Let $e^C = A$: $P = 800 + Ae^{-Kt}$

$500 \overset{set}{=} 800 + Ae^{0}$ ①

① $A = -300$

② $P(t) = 800 - 300e^{-Kt}$

NOTICE P IS INCREASING as $t \to \infty$ since we'll will be subtracting less as t grows.

③ B) $P(t) = 800 - 300e^{-Kt}$

$P(2) = 800 - 300e^{-2K} \overset{set}{=} 700$ ①

$-300e^{-2K} = -100$

$e^{-2K} = \frac{1}{3}$

$-2K = -\ln 3$

$2K = \ln 3$

② $K = \frac{1}{2}\ln 3$

$P(t) = 800 - 300e^{-\frac{t}{2}\ln 3}$

c) $\lim\limits_{t\to\infty} 800 - \frac{300}{e^{\frac{t}{2}\ln 3}}$ ②

$\xrightarrow{} 0$

800 ③

This seems too obvious!

⑥
1993

6 a) $\frac{dP}{dt} = K(800-P)$

② $\int \frac{dP}{800-P} = \int K\,dt$

I don't like this method because $d* < 0$

$-\ln|800-P| = Kt + C$

$800 + P = Ae^{Kt}$

② $P = -800 + Ae^{Kt}$

① $P(0) = 500 \overset{\text{set}}{=} 800 - Ae^{K \cdot 0}$

$800 - A$

$A = 300$

$P(t) = 800 - 300e^{Kt}$ ①

⑤

b) $P(2) = 700$

① → $800 - 300e^{2K} \overset{\text{set}}{=} 700$ ①

$-300e^{2K} = -100$

$e^{2K} = \frac{1}{3}$

$\ln(e^{2K}) = -\ln 3$

$2K = -\ln 3$

③ $K = -\frac{1}{2}\ln 3$

$K = -.549$

$e^{(-\frac{1}{2}\ln 3)t}$

$e^{\ln 3^{-\frac{t}{2}}}$ ③ Simplify

$3^{-\frac{t}{2}}$

④ c)

$\lim_{t \to \infty} 800 - 300e^{(-\frac{1}{2}\ln 3)t}$

$\lim_{t \to \infty} 800 - \frac{300}{3^{\frac{t}{2}}}$

$800 - \frac{300}{\text{large}}$ ①

$\downarrow$
0

$800.$

$$\frac{dP}{dt} = K(800-P)$$

$$\frac{dP}{dt} = -K(P-800)$$

$$\int \frac{dP}{P-800} = \int -K\,dt$$

$$\ln|P-800| = -KT+C$$

$$|P-800| = e^{-Kt+C}$$

$$P-800 = Ae^{-Kt}$$

$$P = 800 + Ae^{-Kt}$$

$$P(0) = 500 \overset{set}{=} 800 + Ae^{0}$$

$$\therefore A = -300$$

a $$P(t) = 800 - 300e^{-Kt}$$

---

b $$P(2) = 800 - 300e^{-2K} \overset{set}{=} 700$$

$$300e^{-2K} = 100$$

$$e^{-2K} = \frac{1}{3}$$

$$-2K = -\ln 3$$

$$K = \frac{1}{2}\ln 3$$

---

c $$\lim_{t\to\infty} 800 - \frac{300}{e^{Kt}}$$

$$800 - 0 = 800$$

5. Let f be the function given by $f(x) = e^{-x}$, and let g be the function given by $g(x) = kx$, where k is the nonzero constant such that the graph of f is tangent to the graph of g.

a) Find the x–coordinate of the point of tangency and the value of k.

b) Let R be the region enclosed by the y–axis and the graphs of f and g. Using the results found in part (a), determine the area of R.

c) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated by revolving the region R, given in part (b), about the x–axis.

6. At time t, $t \geq 0$, the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius. At $t = 0$, the radius of the sphere is 1 and at $t = 15$, the radius is 2. (The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

a) Find the radius of the sphere as a function of t.

b) At what time t will the volume of the sphere be 27 times its volume at $t = 0$?

END OF EXAM

1992

A.P. Calculus AB exam

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e).
(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Let f be the function defined by : $f(x) = 3x^5 - 5x^3 + 2$.
   - a) On what intervals is f increasing?
   - b) On what intervals is the graph of f concave upward?
   - c) Write the equation of each horizontal tangent line to the graph of f .

2. A particle moves along the x–axis so that its velocity at time t, $0 \le t \le 5$, is given by $v(t) = 3(t-1)(t-3)$. At time $t = 2$, the position of the particle is $x(2) = 0$.
   - a) Find the minimum acceleration of the particle.
   - b) Find the total distance traveled by the particle.
   - c) Find the average velocity of the particle over the interval $0 \le t \le 5$.

3. Let f be the function given by $f(x) = \ln\left|\frac{x}{1+x^2}\right|$.
   - a) Find the domain of f.
   - b) Determine whether f is an even function, an odd function , or neither. Justify your conclusion.
   - c) At what values of x does f have a relative maximum or a relative minimum? For each such x, use the first derivative test to determine whether $f(x)$ is a relative maximum or a relative minimum.
   - d) Find the range of f.

4.) Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \le y \le 2\pi$.
   - a) Find $\frac{dy}{dx}$ in terms of y.
   - b) Write an equation for each vertical tangent to the curve.
   - c) Find $\frac{d^2y}{dx^2}$ in terms of y.

1992

1992

① c) $y = 4$
$y = 0$
$y = 2$

② b) total distance traveled 28

③ d) $f(x) \leq -\ln 2$

④ c) $\frac{d^2y}{dx^2} = \frac{\cos y}{(1-\sin y)^3}$

⑤ a) $x = -1$ $k = -e$

⑥ a) $r = (t+1)^{\frac{1}{4}}$

1992
#6

$\frac{dV}{dT}=\frac{K}{R}$ $T=0;\ R=1$ $T=15;\ R=2$ $V=\frac{4}{3}\pi r^3$

⑨ a) $V=\frac{4}{3}\pi r^3$

want r in terms of T we don't have a "T". take Der WRT T. substitute integrate to get T.

$\frac{dV}{dT}=4\pi r^2\frac{dr}{dT}$ ①

② $\frac{K}{r}=4\pi r^2\frac{dr}{dT}$

$\int K\,dT=4\pi\int r^3\,dr$ ①

$KT+C=\pi r^4$

$T=0,\ R=1$ $0+C=\pi$

$C=\pi$ ①

$KT+\pi=\pi r^4$

$T=15,\ r=2$ $15K+\pi=16\pi$ ①

$15K=15\pi$

$K=\pi$ ①

$\pi T+\pi=\pi r^4$

$T+1=r^4$

$(T+1)^{\frac{1}{4}}=r$. ②

⑥ B) New Vol $=\overset{9}{\cancel{27}}\cdot\frac{4}{\cancel{3}}\pi(1)^3$ $T=0,\ r=1$

36π ②

$\frac{4}{3}\pi r^3\overset{\text{set}}{=}36\pi$ ①

$\frac{4}{3}\pi(T+1)^{\frac{3}{4}}=\overset{9}{\cancel{36}}\cancel{\pi}\cdot\frac{3}{4}$

$(T+1)^{3/4}=27$

$(T+1)^{\frac{1}{4}}=3$

$T+1=81$

$T=80$ ②

when T=80 the sphere will have 27 times the ① volume it had when T=0.

1992
#1

$f(x) = 3x^5 - 5x^3 + 2$

WTF $f'(x) > 0$ ①

⑤ a) $f'(x) = 15x^4 - 15x^2$ ①

$15x^2(x^2-1)$

① $15x^2(x-1)(x+1)$

①
f' $+ \quad 0 \quad - \quad \otimes \quad - \quad 0 \quad +$
$-1 \qquad 0 \qquad 1$

① ②
f is increasing for $x < -1$ or $x > 1$

WTF $f'' > 0$ ①

⑥ b) $f''(x) = 60x^3 - 30x$ ①

$30x(2x^2-1)$ ① $\overset{\text{set}}{=} 0$

$x \in \{0, \pm\frac{\sqrt{2}}{2}\}$ ①

$- \quad 0 \quad + \quad 0 \quad - \quad 0 \quad +$ ①
$-\frac{\sqrt{2}}{2} \qquad 0 \qquad \frac{\sqrt{2}}{2}$

f concave up $-\frac{\sqrt{2}}{2} < x < 0$ or
① ②
$x > \frac{\sqrt{2}}{2}$

④ c) horizontal tangents when $f' = 0 = m$ ①

$x \in \{-1, 1\}$. Note $x = 0$ $(0, 2)$ LAZY PI ① or state horizontal tangents occur at max, min pts. AND establish them

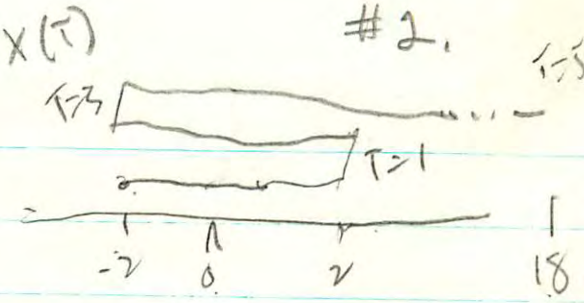
$f(-1) = -3 + 5 + 2 = 4$ $\quad y = 4$ ①

$f(1) = 3 - 5 + 2 = 0$ $\quad y = 0$ ①

$f(0) = \quad = 2$ $\quad y = 2$

The problem asks for horizontal tangent not extremum. at a LPI the tangent line is horizontal.

1992
#2.



$v(t) = 3(\tau-1)(\tau-3)$

$x(2) = 0 \quad v = 3\tau^2 - 12\tau + 9$

$0 \leq \tau \leq 5$

⑥ a) acceleration $= \frac{dv}{dt} \overset{①}{=} 6\tau - 12 = A(\tau)$

$A' = 6$ ①. $\quad A(t) \; \underline{\quad ① \quad +}$ $\overset{\uparrow}{0}$

① acceleration always increasing ∴
min value occurs at END pt $\tau = 0$ ① (increasing from it)
$A(0) = -12$ ∴ min acceleration is -12 (actually deceleration)
①

⑥ b) v ① $\underline{\quad + \; 0 \; - \; 0 \; + \quad}$
$\overset{\uparrow}{0} \quad 1 \quad 3 \quad 5$

$\int_0 v\,dt = x(t)$ ① $\quad \int 3\tau^2 - 12\tau + 9\,dt = \tau^3 - 6\tau^2 + 9\tau + c$

$x(2) = 8 - 24 + 18 + c \overset{\text{Set}}{=} 0$ ① $\quad 8 - 24 + 64 + c0$

$c = -2$

$x(\tau) = \tau^3 - 6\tau^2 + 9\tau - 2$

	1	−6	9	−2
0]	1			−2
1]	1	−5	4	2
3]	1	−3	0	−2
5]	1	−1	4	18

①

$|2-(-2)| + |-2-(2)| + |18-(-2)|$

① $4 + 4 + 20$

TOTAL Dist Traveled 28

③ c) average Velocity is Ave of fctn:

$\frac{1}{b-a}\int_a^b v(\tau)\,dt$ ①

$\frac{1}{5}\int_0^5 3\tau^2 - 12\tau + 9\,dt$ ① Proper fctn

$\frac{1}{5}\left(\tau^3 - 6\tau^2 + 9\tau\right)\Big|_0^5$

$\frac{\tau}{5}\left(\tau^2 - 6\tau + 9\right)$

$\frac{\tau}{5}(\tau-3)^2\Big|_0^5 = 1 \cdot 2^2 = 4.$ ①

$f(x) = \ln\left|\frac{x}{1+x^2}\right|$

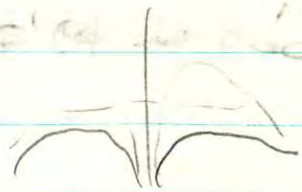
1992
#3

② a) $D_f = ?$ $D_{\ln ✲}$ is ✲ > 0 ① so $\left|\frac{x}{1+x^2}\right| > 0 \ \forall x \neq 0$
$\therefore D_f \ x \neq 0$ ①

④ b) if f is even then $f(-x) = f(x)$ ①

② $\left\{ \begin{array}{l} f(-x) = \ln\left|\frac{-x}{1+(-x)^2}\right| \\ \quad \downarrow \quad = \ln\left|\frac{x}{1+x^2}\right| \\ f(-x) = f(x) \ \therefore f \text{ is Even} \end{array} \right.$ ①

→ so f is reflected in y axis; also f'(x) is odd.



⑥ c) Since f is Even, f'(x) is odd, therefore $f(-x) = -f(x)$
② we need ~~only~~ only Determine interval for f(x) and apply the negative to determine the intervals for $x < 0$.

$x > 0$ $\quad f(x) = \ln\left(\frac{x}{1+x^2}\right)$

$f'(x) = \frac{1+x^2}{x}\left[\frac{1+x^2-2x^2}{(1+x^2)^2}\right]$

① $= \frac{1-x^2}{x(1+x^2)}$

$\rightarrow f' \quad \vdots \quad + \ 0 \ -$ on a line marked $\overset{\wedge}{0}$ and 1

$f'(\text{large}) = \frac{1-\text{large}}{\text{large}} < 0$

so

$f'(x)$ for $x \in (-1, 0) = -f'(x)$ for $x \in (0, 1) = (-)(+) = -$

① $f'(x)$ for $x < -1 = -f'(x)$ for $x > 1 = (-)(-) = +$

f' ① $\quad + \ 0 \ - \ \overset{\wedge}{0} \ + \ 0 \ -$ on a line marked -1, 0, 1

Rel max at $x \in \{-1, 1\}$
② by f' test.
No Rel min since zero not element in D_f.

③ D) $f(-1) = \ln\frac{1}{2} = -\ln 2$
$f(1) = \ln\frac{1}{2} = -\ln 2$

② $\lim_{x \to \pm\infty} f(x) \to -\infty$

$\lim_{x \to 0^\pm} f(x) \to -\infty$

Range of f $\quad f(x) \leq -\ln 2$. ①

1992
#4

$y + \cos y = x + 1 \qquad 0 \le y \le 2\pi$

③ a) $\frac{dy}{dx} = ?$ $\qquad \frac{dy}{dx} - \sin y \overset{\textcircled{1}}{\frac{dy}{dx}} = 1$ ①

$\frac{dy}{dx} = \frac{1}{1 - \sin y}$ ①

⑤ ~~⑧~~ b) Vertical tangent when $\frac{dy}{dx}$ DNE ie when ÷ 0 ①

$1 - \sin y \overset{\text{set}}{=} 0$ ①

$\sin y = 1$

$y \in \left\{\frac{\pi}{2}\right\}$ ①

only 1 vertical tangent

② $y + \cos y = x + 1$

$\frac{\pi}{2} + 0 = x + 1$

$x = \frac{\pi - 2}{2}$

⑦ c) $\frac{dy}{dx} = \frac{1}{1 - \sin y}$

$\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dx}$ ① $\qquad \frac{dy}{dx} = (1 - \sin y)^{-1}$

$\frac{d^2y}{dx} = -(1 - \sin y)^{-2}\left[-\cos y \frac{dy}{dx}\right]$ ② ②

$= \frac{\cos y}{(1 - \sin y)^2} \cdot \frac{1}{1 - \sin y}$

$= \frac{\cos y}{(1 - \sin y)^3}$ ①

$f(x) = e^{-x}$ $g(x) = Kx$ $K \neq 0$

NOTE Y intercept is zero!

1992
#5

⑦ a) ★ $\overset{①}{e^{-x}} \overset{set}{=} Kx$ Y values are = when fctns are tangent,

① $f'(x) = -e^{-x} \overset{set}{=} K = g'(x)$ fctn and tangent have same slope at that point.

Since $e^{-x} = Kx$, then $-e^{-x} = -Kx$

$-K = Kx$ $\qquad -(-e^{-x}) = e^{-x}$ substitute in ★

② $Kx + K = 0$

$K(x+1) = 0$ $\quad K \neq 0$①!, divide by K.

① $x = -1$ x-coord of pt of tangency

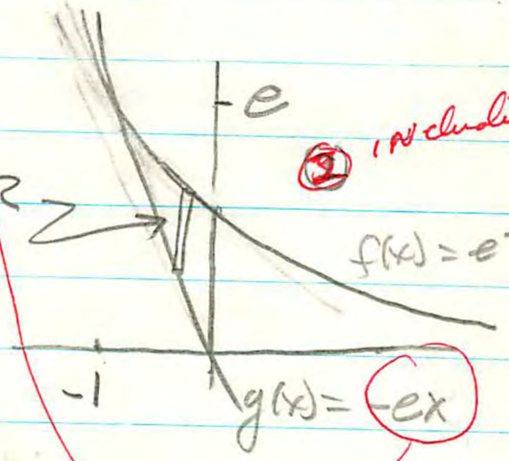
substitute $x = -1$ into ★ $e = -K$ so $K = -e$. ①

④ b) $\displaystyle\int_{-1}^{0} e^{-x} \overset{①}{+} ex \, dx$

$\displaystyle -e^{-x} + \frac{ex^2}{2} \Big|_{-1}^{0} = -1 + 0 - \left[-e + \tfrac{1}{2}e\right]$

$= -1 + \frac{1}{2}e$

①

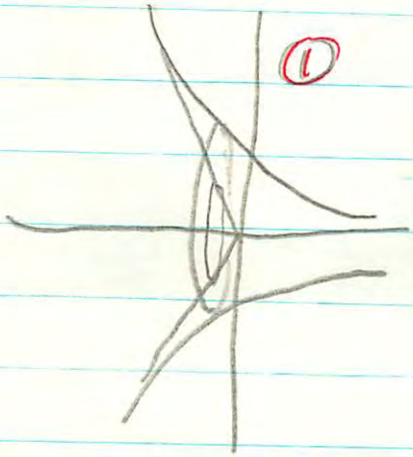


② including Label

④ c) washers. ① $\pi \int r_1^2 - r_2^2 \, dx$

$\displaystyle \pi \int_{-1}^{0} e^{-2x} - (e^2x^2) \, dx$ ① Proper square.

$\displaystyle \pi \int_{-1}^{0} e^{-2x} - e^2x^2 \, dx$ ①



①

42 Exams @ $59 2478

10 @ $5 50

2528

42 Exams @ $60 ~~3120~~

2520 collected

10 @ $5 50 collected

2570

− 2528

$42 Excess.

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$10 used for MAA sample materials for 2nd level MAA.

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REPORT TO AF ACHER
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MATHEMATICS - CALCULUS AB

The following charts indicate the number of AP candidates from your school/district in each fifth of the total candidate group for each category described.

MULTIPLE-CHOICE	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH
DIFFERENTIAL CALCULUS (28 Q.)		6	5	6*	8
INTEGRAL CALCULUS (17 Q.)	1	6	8*	7	3
TOTAL (45 QUESTIONS)		6	6	9*	4
** PERCENT IN EACH FIFTH		24%	24%	36%*	16%
	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH

TOTAL AP GROUPS BY FIFTHS

FREE-RESPONSE	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH
1		6	9*	5	5
2	3	2	9*	6	5
3	2	3	4	10*	6
4	5	6	3*	7	4
5	5	10*	3	4	3
6	5	5	4*	4	7
TOTAL (FREE RESPONSE)	1	7	8*	4	5
** PERCENT IN EACH FIFTH	4%	28%	32%*	16%	20%
	LOWEST FIFTH	SECOND FIFTH	THIRD FIFTH	FOURTH FIFTH	HIGHEST FIFTH

TOTAL AP GROUPS BY FIFTHS

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VACAVILLE HIGH SCHOOL
AP COORD/MR HERB HAYS
100 MONTE VISTA AVENUE
VACAVILLE CA 95688

* = Location of median (50th percentile) for school/district if given category includes 5 or more candidates

** = Percentage of your students in each fifth for the entire section of the exam

Total number of AP candidates in subject for year = 71804

Number of candidates in your group = 25

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CALCULUS AB

SECTION II

Time—1 hour and 30 minutes
Number of problems—6
Percent of total grade—50

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Let f be the function that is defined for all real numbers x and that has the following properties.

   (i) $f''(x) = 24x - 18$

   (ii) $f'(1) = -6$

   (iii) $f(2) = 0$

   (a) Find each x such that the line tangent to the graph of f at $(x, f(x))$ is horizontal.
   (b) Write an expression for $f(x)$.
   (c) Find the average value of f on the interval $1 \le x \le 3$.

---

2. Let R be the region between the graphs of $y = 1 + \sin(\pi x)$ and $y = x^2$ from $x = 0$ to $x = 1$.

   (a) Find the area of R.
   (b) Set up, but do not integrate an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the x-axis.
   (c) Set up, but do not integrate an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

1991

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3. Let f be the function defined by $f(x) = (1 + \tan x)^{\frac{3}{2}}$ for $-\frac{\pi}{4} < x < \frac{\pi}{2}$.

(a) Write an equation for the line tangent to the graph of f at the point where $x = 0$.

(b) Using the equation found in part (a), approximate $f(0.02)$.

(c) Let f^{-1} denote the inverse function of f. Write an expression that gives $f^{-1}(x)$ for all x in the domain of f^{-1}.

---

4. Let f be the function given by $f(x) = \dfrac{|x| - 2}{x - 2}$.

(a) Find all the zeros of f.

(b) Find $f'(1)$.

(c) Find $f'(-1)$.

(d) Find the range of f.

---

5. Let f be a function that is even and continuous on the closed interval $[-3, 3]$. The function f and its derivatives have the properties indicated in the table below.

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$
$f(x)$	1	Positive	0	Negative	-1	Negative
$f'(x)$	Undefined	Negative	0	Negative	Undefined	Positive
$f''(x)$	Undefined	Positive	0	Negative	Undefined	Negative

(a) Find the x-coordinate of each point at which f attains an absolute maximum value or an absolute minimum value. For each x-coordinate you give, state whether f attains an absolute maximum or an absolute minimum.

(b) Find the x-coordinate of each point of inflection on the graph of f. Justify your answer.

(c) In the xy-plane provided below, sketch the graph of a function with all the given characteristics of f.

Note: The xy-plane is provided in the pink test booklet only.

1991

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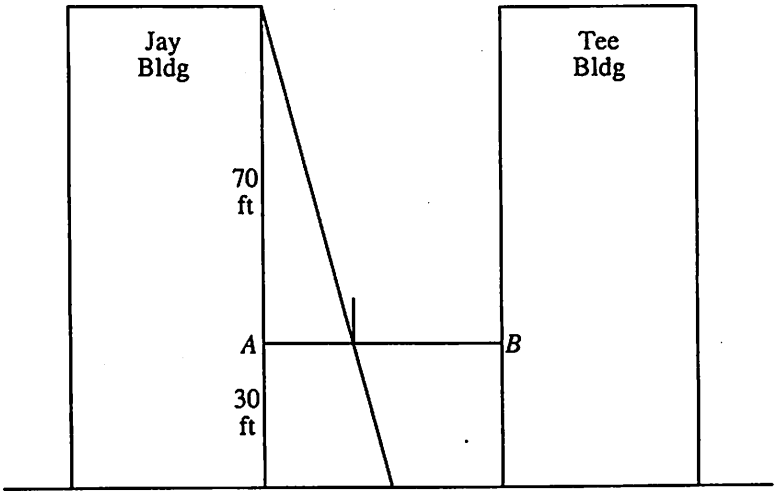


6. A tightrope is stretched 30 feet above the ground between the Jay and the Tee buildings, which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second from point A to point B, is illuminated by a spotlight 70 feet above point A, as shown in the diagram.

   (a) How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the buildings? (Indicate units of measure.)

   (b) How far from point A is the tightrope walker when the shadow of her feet reaches the base of the Tee Building? (Indicate units of measure.)

   (c) How fast is the shadow of the tightrope walker's feet moving up the wall of the Tee building when she is 10 feet from point B? (Indicate units of measure.)

END OF EXAMINATION.

1991

1991

①

i $f''(x) = 24x - 18$

ii $f'(1) = -6$

iii $f(2) = 0$

⑤

a) horizontal tangent when $f' = 0$ ①

$\int f''\,dx = f'$ ①

$\int 24x - 18 = 12x^2 - 18x + \overset{①}{C} = f'(x)$

$f'(1) = 12 - 18 + C \overset{set}{=} -6$

$-6 + C = -6$

$C = 0$ ①

$f'(x) = 12x^2 - 18x \overset{set}{=} 0$

$6x(2x - 3) = 0$

$x = 0$ or $x = \frac{3}{2}$ ①

tangent horizontal when $x \in \{0, \frac{3}{2}\}$

⑤ B) $f(x) = \int f'(x)\,dx$ ①

$= \int 12x^2 - 18x\,dx$

$f(x) = 4x^3 - 9x^2 + C$ ①

$f(2) = 32 - 36 + C \overset{set}{=} 0$ ①

$C = 4$ ①

$f(x) = 4x^3 - 9x^2 + 4$ ①

⑤ c) average value $\frac{1}{b-a}\int f(x)\,dx$ ①

$\frac{1}{3-1}\int_1^3 4x^3 - 9x^2 + 4\,dx$ ①

$\frac{1}{2}\left(x^4 - 3x^3 + 4x\right)\Big|_1^3$ ①

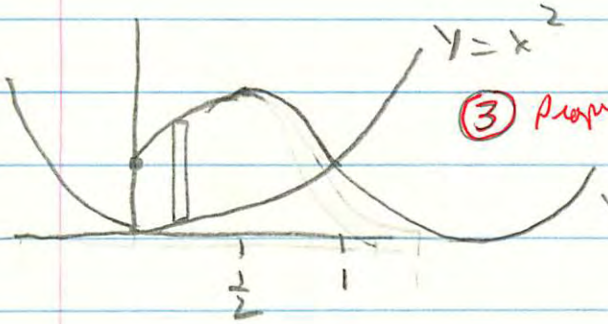
$\frac{1}{2}\left[81 - 81 + 12 - (1 - 3 + 4)\right]$ ①

$\frac{1}{2}\left[12 - 2\right]$

5 Average value ①

1991
②

$Y = 1 + \sin(\pi x) \qquad Y = x^2 \qquad x \in [0, 1]$



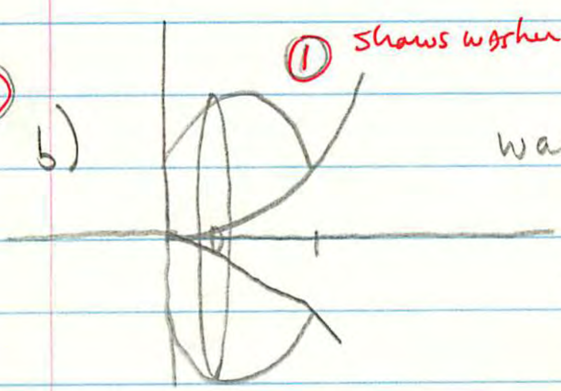
③ Proper graphs & cross sections

⑦ a) $\int_0^1 1 + \sin(\pi x) - x^2\,dx$ ①

$x \overset{①}{-} \overset{①}{\frac{1}{\pi}} \cos\pi x - \frac{x^3}{3}\Big|_0^1 = 1 + \frac{1}{\pi} - \frac{1}{3} - \left(0 - \frac{1}{\pi} - 0\right)$

$\frac{2}{3} + \frac{2}{\pi}$ area. ①

④ b)

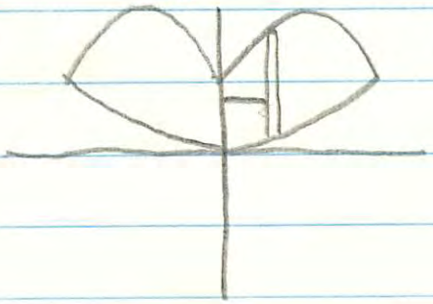


① shows washer

washers $\pi\int r_1^2 - r_2^2\,dx$ ①

$\pi\int_0^1 (1 + \sin(\pi x))^2 - x^4\,dx$ ②

④ c)



① shows shell

Shells $2\pi\int$ radius length dx ①

$2\pi\int_0^1 x\left(1 + \sin(\pi x) - x^2\right) dx$ ②

$f(x) = (1+\tan x)^{3/2} \quad -\frac{\pi}{4} < x < \frac{\pi}{2}$

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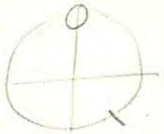
③



This would have been better if evaluation point had been $\frac{\pi}{4}$ *

⑦ a) $f'(x) = \frac{3}{2}(1+\tan x)^{\frac{1}{2}}\sec^2 x \Big|_{x=0}$ ② ①

$\frac{3}{2}(1+0)^{\frac{1}{2}}1^2 = \frac{3}{2} = m$ ①

$f(0) = (1+0)^{3/2} = 1$ ① $(0,1)$ $y-1 = \frac{3}{2}(x-0)$

$y = \frac{3}{2}x + 1$ ②

Not as requested in problem

④ b) $f(x+\Delta x) \approx f(x) + f'(x)\Delta x$ ②

$x = 0$

$\Delta x = .02 \quad f(0+.02) \approx 1 + \frac{3}{2}\cdot\frac{1}{50} = 1 + \frac{3}{100} = 1.03$ ②

$f(0) = 1$

$f'(0) = \frac{3}{2}$

OR

$y = f(x) = \frac{3}{2}x + 1$

$f(.02) = \frac{3}{\cancel{2}}\cdot\frac{\cancel{2}}{100} + 1$

$= 1.03$

④ c) $x = (1+\tan z)^{3/2}$ ①

$x^{\frac{2}{3}} = 1 + \tan z$

$\tan z = x^{\frac{2}{3}} - 1$

$z = \tan^{-1}(x^{\frac{2}{3}} - 1) = f^{-1}(x)$ ③

* $\frac{3}{2}(1+1)^{\frac{1}{2}}(\sqrt{2})^2$ $\quad f(\frac{\pi}{4}) = (1+1)^{\frac{3}{2}}$

$= 2\sqrt{2}$

$3\sqrt{2}\cdot 2$

$6\sqrt{2} = m \quad (\frac{\pi}{4}, 2\sqrt{2})$

maybe not....

$y - 2\sqrt{2} = 6\sqrt{2}(x - \frac{\pi}{4})$

$= 6\sqrt{2}x - 6$

$f(x) = \frac{|x|-2}{x-2}$

1991

④

④ a) $f(x) = \begin{cases} \frac{-x-2}{x-2} & \text{if } x<0 \\ 1 & \text{if } x \geq 0,\ x \neq 2 \end{cases}$ If $x<0$ $f(x) = \frac{-x-2}{x-2}$

$\frac{-x-2}{x-2} \overset{\text{set}}{=} 0$ $x=-2$ is only zero of $f(x)$.

pts for quality of argument

$-x-2=0$

$x=-2$

③ b) $f'(1) = ?$

Since $x>0$ $f(x)=1$

$f'(x) = 0 \therefore f'(1) = 0$

④ c) $f'(-1) = ?$ Since $x<0$ $f(x) = \frac{-x-2}{x-2}\ \begin{matrix} u \\ v \end{matrix}$

$f'(x) = \frac{(x-2)(-1) - (-x-2)(1)}{(x-2)^2}$

$= \frac{-x+2+x+2}{(x-2)^2} = \frac{4}{(x-2)^2}\Big|_{x=-1} = \frac{4}{9}$

④ d) $x = \frac{-z-2}{z-2}$ conclusion Range of f

$-1 < f(x) \leq 1$

$xz - 2x = -z - 2$

$(x+1)z = 2x-2$

$z = \frac{2x-2}{x+1} = f^{-1}(x)$ $x<0$ $D_{f^{-1}}$ $x \neq -1$ ∴ Range f $f(x) \neq -1$

$$x-2 \overline{\smash{)}\,-x-2}^{\,-1}$$
$-x+2$
-4

$f(x) = -1 - \frac{4}{x-2}$ $x<0$

$= -1 + \frac{-4}{x-2}$ $\frac{\text{neg}}{\text{neg}} = +$

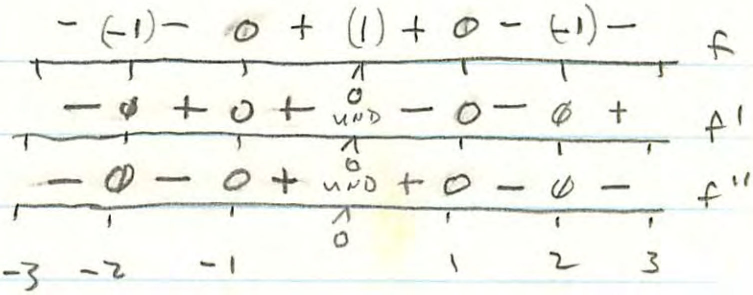
∴ $f(x) > -1$ since $\frac{-4}{x-2} > 0$.

$\lim_{x \to -\infty} f(x) = \frac{-1-\frac{2}{x}}{1-\frac{2}{x}} = -1$

$\lim_{x \to 0^-} f(x) = \frac{0-2}{0-2} = +1$

1991
#5

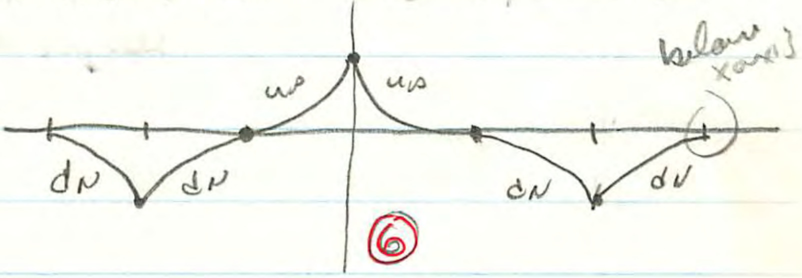
Notice: The "trick" is that f reflects across x = 0 not f' but f" would. f' is odd and reflects as $-f'(x)$



⑤ a) Abs max $x = 0$

increases to it and decreases from it.

Rel max at $x = -3$ (dec from it) and $x = 3$ increases to it. Since $f(x) < 0$ for x near ±3, these are not abs max



⑥

Abs min at $x = -2$

dec to it, inc from it

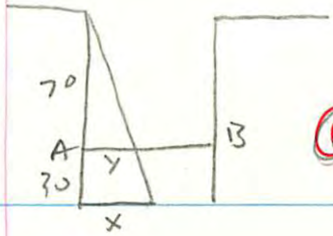
Likewise $x = 2$

④ b) PI occur when there is a change in concavity.

$x = -1$ dN → up ; $x = 1$ up → dN

c) Pict is drawn above.

1991
#6



⑥ A) $\frac{dx}{dt} = 7$ ①

$\frac{dy}{dt} = 2$ ①

$\frac{y}{70} = \frac{x}{100}$ ①

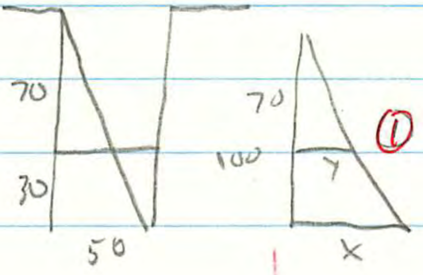
$\frac{10}{7}y = x$

$\frac{10}{7}\frac{dy}{dt} = \frac{dx}{dt}$ ①

$\frac{20}{7} = \frac{dx}{dt}$ ①

the shadow is moving to the right at $\frac{20}{7}$ f/sec ①

③ B)



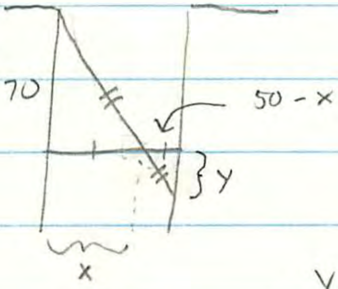
①

$\frac{Y}{70} = \frac{x}{100}$ ①

$Y = 70\left(\frac{5\cancel{0}}{10\cancel{0}}\right)\frac{1}{2}$

$Y = 35$ ①

⑥ C)



$\frac{Y}{70} = \frac{50-x}{x}$ ②

$xY = 3500 - 70x$

$Y\frac{dx}{dt} + x\frac{dY}{dt} = -70\frac{dx}{dt}$ ① $x = 40$

$\frac{y}{70} = \frac{10}{40}$

$Y = \frac{35}{2}$ ①

$\cancel{\frac{35}{2}\cdot 2} + 40\frac{dY}{dt} = -70(2)$

$\frac{dY}{dt} = \frac{-175}{40} = -\frac{35}{8}$ ①

the shadow is moving up the side of the building at

$\frac{35}{8}$ fT/sec. ①

CALCULUS AB
SECTION II

Time—1 hour and 30 minutes
Number of problems—6
Percent of total grade—50

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. A particle, initially at rest, moves along the x-axis so that its acceleration at any time $t \geq 0$ is given by $a(t) = 12t^2 - 4$. The position of the particle when $t = 1$ is $x(1) = 3$.

   (a) Find the values of t for which the particle is at rest.

   (b) Write an expression for the position $x(t)$ of the particle at any time $t \geq 0$.

   (c) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

---

2. Let f be the function given by $f(x) = \ln\left(\frac{x}{x-1}\right)$.

   (a) What is the domain of f?

   (b) Find the value of the derivative of f at $x = -1$.

   (c) Write an expression for $f^{-1}(x)$, where f^{-1} denotes the inverse function of f.

---

3. Let R be the region enclosed by the graphs of $y = e^x$, $y = (x-1)^2$, and the line $x = 1$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

---

4. The radius r of a sphere is increasing at a constant rate of 0.04 centimeters per second. (Note: The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

   (a) At the time when the radius of the sphere is 10 centimeters, what is the rate of increase of its volume?

   (b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?

   (c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?

1990

GO ON TO THE NEXT PAGE

5. [illegible] f be the function defined by $f(x) = \sin^2 x - \sin x$ for $0 \le x \le \frac{3\pi}{2}$.

(a) Find the x-intercepts of the graph of f.

(b) Find the intervals on which f is increasing.

(c) Find the absolute maximum value and the absolute minimum value of f. Justify your answer.

---

6. Let f be the function that is given by $f(x) = \dfrac{ax + b}{x^2 - c}$ and that has the following properties.

(i) The graph of f is symmetric with respect to the y-axis.

(ii) $\lim\limits_{x \to 2^+} f(x) = +\infty$

(iii) $f'(1) = -2$

(a) Determine the values of a, b, and c.

(b) Write an equation for each vertical and each horizontal asymptote of the graph of f.

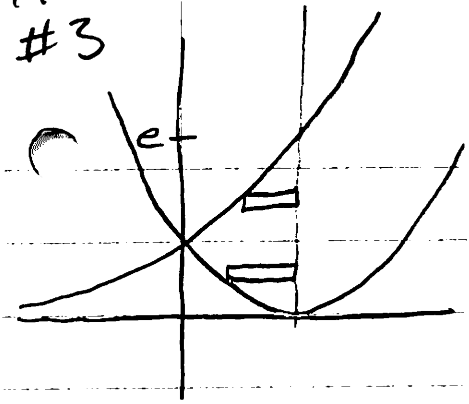
(c) Sketch the graph of f in the xy-plane provided below.

Note: The xy-plane is provided in the pink test booklet only.

END OF EXAMINATION.

1990

1990
#3



A) $y = e^x \therefore \ln y = x$

$y = (x-1)^2 \quad \pm\sqrt{y} + 1 = x$

choose $-\sqrt{y}+1$ to get ~~Left~~ Left side of Parabola.

$1 - \sqrt{y} = x$

$$\int_0^1 (1-(1-\sqrt{y})\,dy + \int_1^e 1 - \ln y\,dy$$

$$\int_0^1 \sqrt{y}\,dy + \int_1^e dy - \int_1^e \ln y\,dy$$

$u = \ln y \quad v = y$

$du = \frac{1}{y}dy \quad dv = dy$

$$\frac{2y^{\frac{3}{2}}}{3}\Big|_0^1 + y\Big|_1^e - y\ln y + y\Big|_1^e$$

$y\ln y - \int dy$

$y \ln y - y$

$$\frac{2}{3} + e - 1 - \cancel{e} + \cancel{e} - 0 - 1 = e - \frac{4}{3}$$

B) Shells use lengths from Part A. multiply by radius, y, and 2π to revolve.

$$2\pi\int_0^1 y\sqrt{y}\,dy + 2\pi\int_1^e y(1-\ln y)\,dy$$

$u = \ln y \quad v = \frac{y^2}{2}dy$

$du = \frac{1}{y}dy \quad dv = y\,dy$

$$2\pi\left[\int_0^1 y^{\frac{3}{2}}dy + \int_1^e y\,dy - \int_1^e y\ln y\,dy\right]$$

$\frac{y^2}{2}\ln y - \int \frac{y}{2}dy$

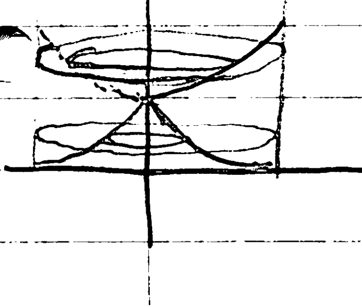
$$2\pi\left[\frac{2y^{\frac{5}{2}}}{5}\Big|_0^1 + \frac{y^2}{2}\Big|_1^e - \frac{y^2}{2}\ln y + \frac{y^2}{4}\Big|_1^e\right.$$

$$2\pi\left[\frac{2}{5} + \cancel{\frac{e^2}{2}} - \frac{1}{2} - \cancel{\frac{e^2}{2}} + \frac{e^2}{4} + 0 - \frac{1}{4}\right]$$

$$2\pi\left[\frac{e^2}{4} + \frac{2}{5} - \frac{3}{4}\right] = 2\pi\left[\frac{e^2}{4} + \frac{8-15}{20}\right]$$

$$2\pi\left[\frac{e^2}{4} - \frac{7}{20}\right] = \frac{\pi e^2}{2} - \frac{7\pi}{10}$$

c) wAshers



$$\pi\int_0^1 1^2 - (1-\sqrt{y})^2\,dy + \pi\int_1^e 1^2 - (\ln y)^2\,dy$$

1990
#6

$f(x) = \frac{ax+b}{x^2-c}$

i f symmt WRT Y axis
∴ f is even fctn.
$f(-x) = f(x)$

ii $\lim_{x \to 2^+} f(x) \to +\infty$

iii $f'(1) = -2$

7 A) Since $f(x) = f(-x)$

$a = 0$

② $x^2 = (-x)^2$ so denom doesn't change. $ax+b$ would becom $-ax+b$ if $a \neq 0$. can't happen ∴ $a = 0$

includes justification

$f(x) = \frac{b}{x^2-c}$ $c = 4$ $\lim_{x \to 2^+} f(x) \to +\infty$ ② so $\frac{b}{x^2-4} \to \infty$ as $x \to 2$

$f(x) = \frac{b}{x^2-4} = b(x^2-4)^{-1}$

$f'(x) = \frac{-b\,2x}{(x^2-4)^2} = \frac{-2bx}{(x^2-4)^2}$ ①

$f'(1) = \frac{-2b}{-9} \overset{\text{set}}{=} -2$ ①

$b = 9$ ① $a = 0,\ b = 9,\ c = 4$

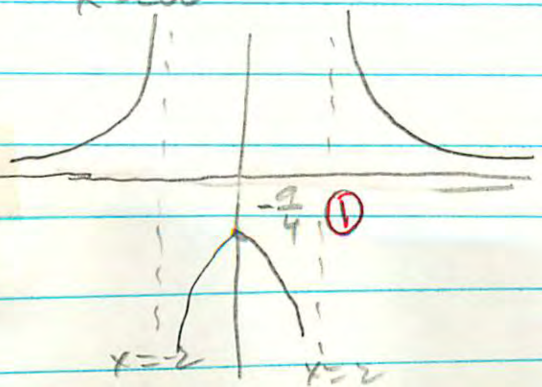
5 B) Vertical asymptote from ① non reducible Domain restrictions.

$x^2 - 4 \overset{\text{set}}{=} 0$ $x = 2$ $x = -2$ ②

horizontal asymptote $\lim_{x \to \infty} f(x) = $ asymptote. ①

$\lim_{x \to \pm\infty} \frac{9}{x^2-4} = 0$ ∴ $Y = 0$ horizontal asymptote. ①

3 C)



Reflects ①

$\lim_{x \to 2} f(x) \to +\infty$ ①

1990
#1

$a(t) = 12\tau^2 - 4$ $\tau = 1,\ x(1) = 3$

7 A) Particle at rest when velocity is zero $\tau \geq 0$

① $v(\tau) = \int a(\tau)$

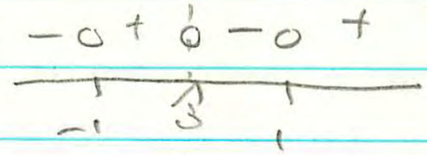
②

$v(\tau) = \int 12\tau^2 - 4\, dt$ "initially at rest" means $v(0) = 0$

$= 4\tau^3 - 4\tau + c$ $\therefore\ c = 0$ ①

$v(\tau) = 4\tau^3 - 4\tau \overset{\text{Set}}{=} 0$ ①

$4\tau(\tau^2 - 1) = 0$



② Particle is at rest initially, ie $\tau = 0$ and when $\tau = 1$.
$\tau = -1$ is not included in the Domain

4 B) $x(\tau) = \int v(\tau)$ ①

$x(\tau) = \int 4\tau^3 - 4\tau\, dt$

$x(\tau) = \tau^4 - 2\tau^2 + c$ ①

$x(1) = 1 - 2 + c \overset{\text{Set}}{=} 3$

$c = 4$ ①

$x(\tau) = \tau^4 - 2\tau^2 + 4$ ①

$x(0) = 4$

$x(1) = 3$

$x(2) = 16 - 8 + 4 = 12$

4 c) Since particle travels left 1st then right, we ②
need to add "left" distance and right distance
to get "total" distance

Left distance = from 4 to 3 = 1

Right distance = from 3 to 12 = 9

total distance = 10 ②

1990
#2

$f(x) = \ln\left(\frac{x}{x-1}\right)$

5 A) Domain of $\ln \star$, $\star > 0$ ①　　$\frac{x}{x-1} > 0$ ②　　$+ \ 0 \ - \ 0 \ +$
$0 \quad 1$

②
D_f　$x < 0$ OR $x > 1$

① CANNOT "split" ln since $x<0$ is allowed, ie $\ln x - \ln(x-1)$ etc,

5 B) $f'(-1)$　　$f(x) = \ln\left(\frac{x}{x-1}\right)$ $\overset{a}{v}$

$f'(x) = \frac{x-1}{x}\left[\frac{(x-1)-x}{(x-1)^2}\right]$ ②

$= \frac{x-1}{x}\left[\frac{-1}{(x-1)^2}\right] = \frac{-1}{x(x-1)}\Big|_{x=-1}$ ②

$\frac{-1}{-1(-2)} = \frac{-1}{2}$

5 C) $f(x) = \ln\left(\frac{x}{x-1}\right)$

$x = \ln\left(\frac{z}{z-1}\right)$ ①

$e^x = \frac{z}{z-1}$ ①

$e^x z - e^x = z$

$z(e^x - 1) = e^x$

$z = \frac{e^x}{e^x - 1}$

$f^{-1}(x) = \frac{e^x}{e^x - 1}$ ②

Verify f^{-1} by $f^{-1}(f(x)) = x$

$f^{-1}(f(x)) = \frac{e^{\ln\left(\frac{x}{x-1}\right)}}{e^{\ln\left(\frac{x}{x-1}\right)} - 1}$

$= \frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1}$

$\frac{\frac{x}{\cancel{x-1}}}{\frac{x-x+1}{\cancel{x-1}}}$

①

x Verified.

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#4

$\frac{dr}{dt} = .04$ c/sec. $V = \frac{4}{3}\pi r^3$

4 A) $\left.\frac{dv}{dt}\right|_{r=10} = ?$ ① $\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$ ②

$= 400\pi\left(\frac{4}{100}\right)$

$= 16\pi$ c³/sec. ①

6 B) Cross section through center is a circle radius r. ①

$V = \frac{4}{3}\cancel{\pi} r^3 \overset{set}{=} \overset{9}{\cancel{36}}\cancel{\pi}\,\frac{3}{\cancel{4}}$

$r^3 = 27$ ②

$r = 3$ when Vol $= 36\pi$

$\left.\frac{dA}{dt}\right|_{r=3} = ?$ ①

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ ①

$= 2\pi \cdot 3\left(\frac{4}{100}\right) = .24\pi = \frac{6\pi}{25}$ ①

5 C) $\frac{dv}{dt} \overset{set}{=} \frac{dr}{dt}$ ②

$4\pi r^2 \frac{dr}{dt} = \frac{dr}{dt}$ ①

$r^2 = \frac{1}{4\pi}$ Reject Negative Solution

$r = \frac{1}{2\sqrt{\pi}}$ ②

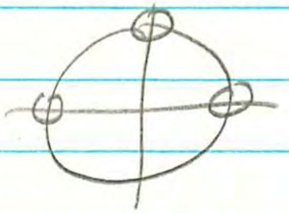
1990
#5

$f(x) = \sin^2 x - \sin x \quad 0 \le x \le \frac{3\pi}{2}$

3 A) $\sin^2 x - \sin x \overset{set}{=} 0$

$\sin x(\sin x - 1) = 0$ ①

$\sin x = 0 \quad \sin x = 1$



①

① x intercepts when $f(x) = 0 \quad x \in \{0, \frac{\pi}{2}, \pi\}$

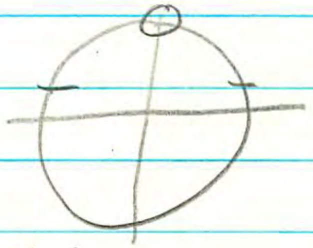
6 B) f increasing when $f'(x) > 0$ ①

$f(x) = \sin^2 x - \sin x$

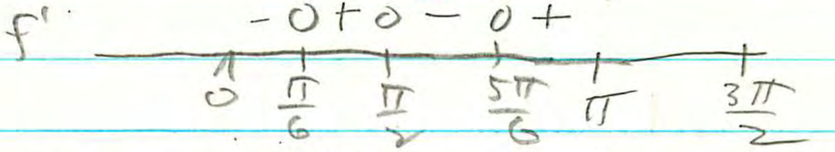
$f'(x) = 2\sin x \cos x - \cos x$ ②

$\cos x(2\sin x - 1) \overset{set}{=} 0$ ①

$\cos x = 0 \quad \sin x > \frac{1}{2}$



①



① f increasing: $\frac{\pi}{6} < x < \frac{\pi}{2}$ OR $\frac{5\pi}{6} < x \le \frac{3\pi}{2}$

6 c) $f(0) = 0$ ENDPT dec from it max

$f(\frac{\pi}{6}) = -\frac{1}{4}$ min f' test

all 5 values ① $f(\frac{\pi}{2}) = 0$ max f' test — Justify ③

$f(\frac{5\pi}{6}) = -\frac{1}{4}$ min f' test

$f(\frac{3\pi}{2}) = 2$ ENDPT INC to it max

① Absolute max value 2

① Absolute min value $-\frac{1}{4}$ Both are Abs min.

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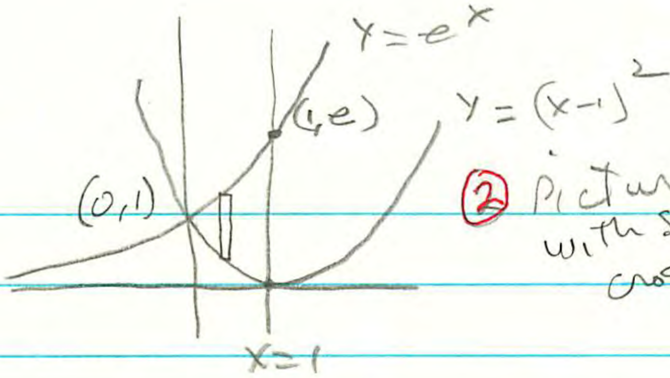
#3



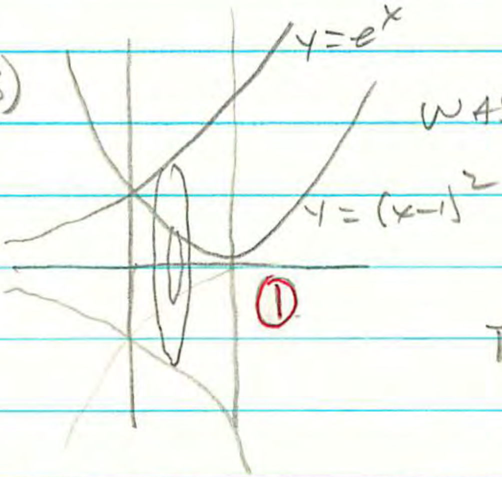
② Picture with sample cross sections

5 A) $\int_0^1 e^x-(x-1)^2\,dx$ ①

$\int_0^1 e^x-x^2+2x-1\,dx = e^x-\frac{x^3}{3}+x^2-x\Big|_0^1$

$\overset{①}{e-\frac{1}{3}+1-1-1} = \overset{①}{e-\frac{4}{3}}$

7 B)



①

washers ① $\pi\int r_1^2-r_2^2$

$\pi\int_0^1 e^{2x} \overset{①}{-} (x-1)^4\,dx$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1 ①

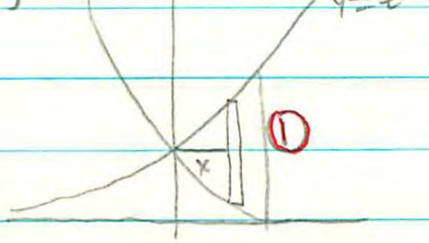
$\pi\int_0^1 e^{2x}\overset{①}{-}(x^4-4x^3+6x^2-4x+1)\,dx$

$\pi\int_0^1 e^{2x}-x^4+4x^3-6x^2+4x-1 =$

$\pi\left[\frac{1}{2}e^{2x}-\frac{x^5}{5}+x^4-2x^3+2x^2-x\right]_0^1$

$\pi\left[\frac{1}{2}e^2-\frac{1}{5}\cancel{+1}\overset{①}{\cancel{-2+2-1}}-\frac{1}{2}\right] = \overset{①}{\frac{\pi}{2}e^2-\frac{\pi}{10}}$

3 C)

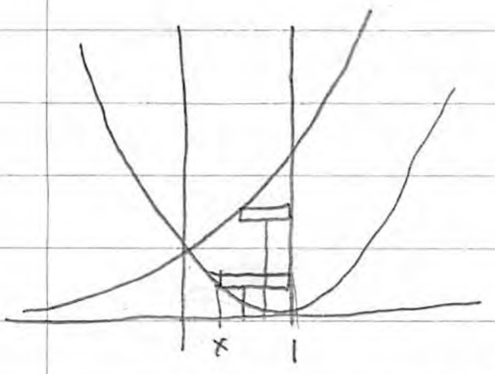


① Shells $2\pi\int$ radius · length of cross section.

①

$2\pi\int_0^1 x(e^x-x^2+2x-1)\,dx$ ①

1990
3B



$y = (x-1)^2$ $\quad$ $y = e^x$

$dy = 2(x-1)\,dx$ $\quad$ $dy = e^x\,dx$

B) Shells — in x

$$2\pi\int_{x=0}^{x=1} (x-1)^2(1-x)\,dy + 2\pi\int_{x=0}^{x=1} e^x(1-x)\,dy$$

$$2\pi\int_0^1 (x-1)^2(1-x)(2(x-1))\,dx + 2\pi\int_0^1 e^x(1-x)e^x\,dx$$

$u = x \quad v = \frac{1}{2}e^{2x}$

$du = dx \quad dv = e^{2x}dx$

$$-4\pi\int_0^1 (x-1)^4\,dx + 2\pi\int_0^1 e^{2x}(1-x)\,dx$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$$-4\pi\int_0^1 x^4 - 4x^3 + 6x^2 - 4x + 1\,dx + 2\pi\int_0^1 e^{2x}\,dx - 2\pi\int_0^1 xe^{2x}\,dx$$

$$-4\pi\left(\frac{x^5}{5} - x^4 + 2x^3 - 2x^2 + x\right)\Big|_0^1 + \frac{2\pi e^{2x}}{2}\Big|_0^1 - \cancel{2}\pi\frac{1}{\cancel{2}}xe^{2x} + 2\pi\frac{1}{2}\int e^{2x}\,dx$$

$$\pi e^{2x} \quad -\pi xe^{2x} + \frac{\pi}{2}e^{2x}\Big|_0^1$$

Need to change so I get $+\frac{4\pi}{5}$

$$-4\pi\left(\frac{1}{5} - 1 + 2 - 2 + 1\right) + \cancel{\pi e^2} - \cancel{\pi e^2} + \frac{\pi}{2}e^2 - \pi - \frac{\pi}{2}$$

$$\to -\frac{4\pi}{5}$$

$$\frac{\pi}{2}e^2 - \frac{3\pi}{2}$$

$$\frac{\pi}{2}e^2$$

CALCULUS BC

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. A particle starts at time $t = 0$ and moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = (t - 1)^3(2t - 3)$.

   (a) Find the velocity of the particle at any time $t \geq 0$.

   (b) For what values of t is the velocity of the particle less than zero?

   (c) Find the value of t when the particle is moving and the acceleration is zero.

---

2. Let R be the region in the xy-plane between the graphs of $y = e^x$ and $y = e^{-x}$ from $x = 0$ to $x = 2$.

   (a) Find the volume of the solid generated when R is revolved about the x-axis.

   (b) Find the volume of the solid generated when R is revolved about the y-axis.

---

3. Let $f(x) = 12 - x^2$ for $x \geq 0$ and $f(x) \geq 0$.

   (a) The line tangent to the graph of f at the point $(k, f(k))$ intercepts the x-axis at $x = 4$. What is the value of k?

   (b) An isosceles triangle whose base is the interval from $(0, 0)$ to $(c, 0)$ has its vertex on the graph of f. For what value of c does the triangle have maximum area? Justify your answer.

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GO ON TO THE NEXT PAGE

4. Let R be the region inside the graph of the polar curve $r = 2$ and outside the graph of the polar curve $r = 2(1 - \sin\theta)$.

(a) Sketch the two polar curves in the xy-plane provided below and shade the region R.

Note: The xy-plane is provided in the pink test booklet only.

(b) Find the area of R.

---

5. Let f be the function defined by $f(x) = \frac{1}{x - 1}$.

(a) Write the first four terms and the general term of the Taylor series expansion of $f(x)$ about $x = 2$.

(b) Use the result from part (a) to find the first four terms and the general term of the series expansion about $x = 2$ for $\ln|x - 1|$.

(c) Use the series in part (b) to compute a number that differs from $\ln\frac{3}{2}$ by less than 0.05. Justify your answer.

---

6. Let f and g be continuous functions with the following properties.

(i) $g(x) = A - f(x)$ where A is a constant

(ii) $\int_1^2 f(x)dx = \int_2^3 g(x)dx$

(iii) $\int_2^3 f(x)dx = -3A$

(a) Find $\int_1^3 f(x)dx$ in terms of A.

(b) Find the average value of $g(x)$ in terms of A, over the interval [1, 3].

(c) Find the value of k if $\int_0^1 f(x + 1)dx = kA$.

END OF EXAMINATION.

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Advanced Placement Examination
CALCULUS BC
SECTION II

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CALCULUS BC

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

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   (b) Find the volume of the solid generated when R is revolved about the y-axis.

---

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GO ON TO THE NEXT PAGE

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END OF EXAMINATION.

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